ASIAC

Report No. 187.1A

INTEGRATED DESIGN AND MANUFACTURING METHODS

VOLUME I - AIRFOIL SPLINE ANALYSIS

FOR DESIGN AND MANUFACTURING

August 1987

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OPERATED FOR THE FLIGHT DYNAMICS LABORATORY
BY ANAMET LABORATORIES, INC.

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Operated for the Flight Dynamics Laboratory by Anamet Laboratories, Inc.

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The engineering coordinator for this effort was William J. Craft (Associate Dean, School of Engineering). The principal investigators were Eui H. Park (Assistant Professor, Department of Industrial Engineering) and Samuel Owusu-Ofori (Assistant Professor, Department of Mechanical Engineering). Co-investigators were Steven H. Y. Lai (Assistant Professor, Mechanical Engineering), Celestine A. Ntuen (Assistant Professor, Industrial Engineering), Chin-Sheng Chen (Assistant Professor, Industrial Engineering), and Mohammed Belkhiter (Electronics Engineering-Data Processing, AMP, Westinghouse Electric Corporation, Winston Salem, NC).

The report on this investigation is presented in two volumes. Volume I, presented here, discusses airfoil spline analysis for design and manufacturing, and was authored by Samuel Owusu-Ofori and Steven H. Y. Lai. Volume II discusses new heuristic and artificial intelligence approaches to production scheduling in flexible manufacturing systems and was authored by Eui H. Park, Chin-Sheng Chen, and Celestine A. Ntuen.

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TASK A: AIRFOIL SPLINE ANALYSIS FOR DESIGN AND MANUFACTURING

ABSTRACT

Spline geometries representing airfoil shapes have been found to present difficulties in manufacturing compatibility with design. This work highlights these problems and seeks to find a solution to the problem of profile matching. A spline geometry system has been developed to simulate the design iteration process with manufacturability as the criterion. The computational algorithm and design procedures are documented. The report provides a study of the effect of design knots on the manufactured profile and also a procedure to integrate design and manufacturing data for the production of spline profiles such as the turbine blade section.

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INTRODUCTION

Several techniques have been developed in recent years for the description and the design of complex geometries and surfaces. Designers normally obtain the design points for a complex geometry through a series of cascaded tests or from an existing database. For splines, the geometry is fit to the design points to obtain the design profile. A very common way to do this is to use the French curve or flexi-curve to strategically draw a smooth curve through a given set of points known as the knots. The appropriate curve tends to minimize the strain energy at the knots [1,2]. The turbine blade section is an example of a typical spline profile. Minor changes in the profile of a turbine blade can cause a substantial variation in the blade loss coefficient particularly, when the turbine is operating away from the design point [3].

Since the designed profile of a turbine blade cannot be described by a closed-form mathematical equation, the designer supplies the manufacturing engineer with a number of control points from the design profile and a few physical parameters of the section. In order to provide these points, the design curve is usually sampled by various techniques. Most of the time, the points are chosen for simplicity. Sometimes, the choice of the control points is based on equal arc lengths or equal spacing along a predetermined axis. This enables the storage of the design profile as only a set of control points resulting in an economical use of computer memory. However, the procedure for the regeneration of the design profile for a machining process or the production of dies for forging is different. The data obtained from the design process has to be synthesized by the manufacturing system to produce the required data. This can result in many undesirable variations in the slopes, and curvatures of the profile as well as the geometric properties like area, moment of inertia, and the location of the centroid. When

this happens, additional manufacturing processes are employed to bring the manufactured part to the design specification, thus increasing the production cost. Airfoil shapes, particularly, have therefore been found to present some difficulties in manufacturing compatibility with design tolerances. In the late 1960's and early 1970's, the Automatically Programmed Tools (APT) programming language was the most advanced and comprehensive commercially available language for the programming of Numerically Controlled Machines. The Tabulated Cylinder(TABCYL) definition in APT [4] has been the most commonly used splining routine for CNC programming. Hence the TABCYL definition has been used almost exclusively by turbine blade manufacturers for the definition of the turbine blade profile.

The above discussions indicate that there is the need to integrate the design and manufacturing activities in order to reduce the discrepancies between design requirements and the manufacturing of complex geometries such as splines. Since iterative procedures are generally used to check the manufactured result against the expected design geometry, it is the purpose of this work to simulate this iterative process so that with the proper selection of knots, a turbine blade may be manufactured to match the design profile using a single CNC machining operation.

The primary objectives of this work are three-fold. These are (i) to investigate the effect of the choice of the number of points (knots) as well as their location on the profile of a typical turbine blade, (ii) to define the turbine blade design and manufacturing criteria to obtain the best choice of points (knots) and locations and (iii) to demonstrate the developed splining technique as an integrated prototype system for the design and manufacture of the turbine blade section.

In order to spline the data provided, the spline function as derived by Greville [5] was used. This function is summarized and discussed in the next section. The derivation of the manufacturing data knowing the spline geometry and the specified tolerance is

discussed in the following section. The remainder of the report consists of the comparisons between the design and the manufacturing data and the description of the methodology used to perform the iteration process for the profile matching of the turbine blade sections. The data used in this work was provided by the Advanced Manufacturing Planning Group of Westinghouse Electric Corporation (Winston-Salem, NC), manufacturers of turbine components.

CHAPTER II

MATHEMATICAL DISCUSSION

The fitting of a spline geometry may be thought of mathematically by considering the interpolation between n data points or knots (X_i, Y_i) as in Figure 1 where i= 1, 2, 3,.....,n so that the resulting curve is continuous in position, slope, and curvature at each knot. In order to do this, a function g(x) with continuous derivatives of orders 1, 2, ...,k when 1< k< n must be defined [5].

For the values of k<n, a polynomial s(x) of order 2k-1 can be found for any interval (X_i, X_{i+1}) which satisfies the continuity conditions [5-8]. Hence for k< n, there is a unique solution which is a piecewise function for any given interval. The polynomial s(x) is known as the natural spline function between two adjacent points. Thus the polynomial arcs s(x) that make up the graph of the function g(x) join "smoothly" so that the polynomial that represents g(x) to the left of X_i and the one that represents g(x) to the right of X_i have the same ordinate and the same values of the derivatives of order 1, 2, 3,.., 2k-2 [4,9,10].

2.1 The Design Spline

The third degree spline has been found to approximate the behavior of the mechanical spline used by draftsmen and hence has been used extensively by designers, especially on CAD systems. This spline has been termed as the "cubic-spline" or simply the "C-spline". Between each interval (X_i, X_{i+1}), the natural spline s(x) is given by some third degree polynomial with linear second derivative s"(x). This means that if the values of $s''(X_i)$ and $s''(X_{i+1})$ are known, the intermediate values may be obtained with linear interpolation.

Using Newton's divided difference interpolation formula between

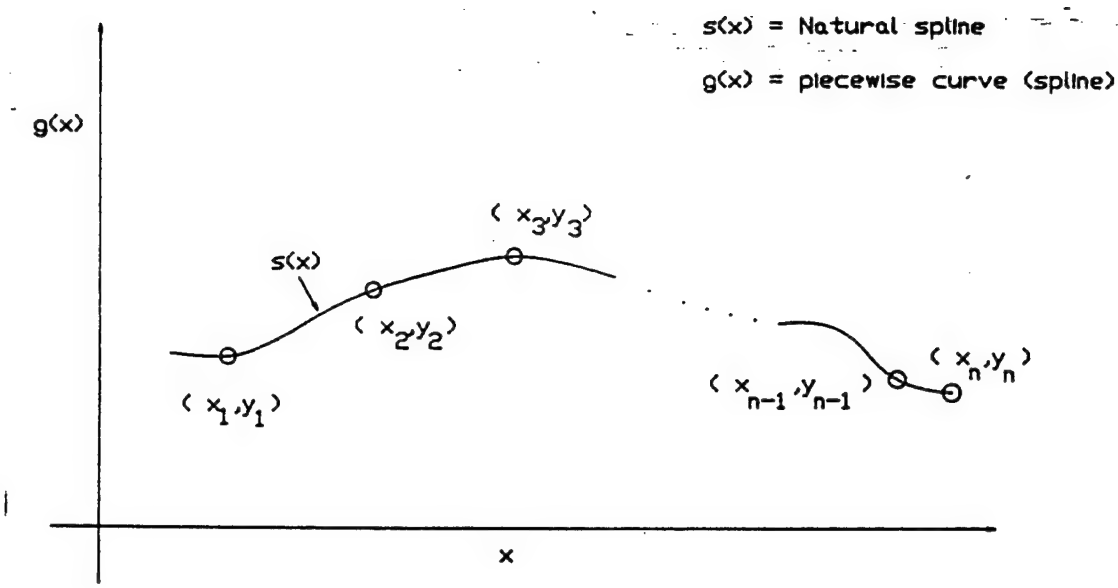


Fig. 1 : Definition of Spline

two adjacent knots (X_i, X_{i+1}) and noting that $s(X_i)=Y_i$, and $s(X_{i+1}) = Y_{i+1}$, the value of the polynomial may be found as follows:

$$s(X) = s(X_i) + (X-X_i).s(X_i,X_{i+1}) + (X-X_i)(X-X_{i+1}).s(X,X_i,X_{i+1}) \tag{1}$$

where

$$s(X_i,X_{i+1}) = s'(X_i) + 1/2(X-X_i).s''(X_i) + 1/6(X-X_i)^2 .s''(X_i,X_{i+1}) \tag{2}$$

and

$$s(X,X_i,X_{i+1}) = 1/2.s''(X_i) + 1/6[(X-X_i)+(X_{i+1}-X_i)].s''(X_i,X_{i+1}) \tag{3}$$

The linearity of $s''(X)$ between (X_i,X_{i+1}) can be represented as:

$$s''(X) = s''(X_i) + (X-X_i).s''(X_i,X_{i+1}) \tag{4}$$

Thus $s(X,X_i,X_{i+1})$ becomes:

$$s(X,X_i,X_{i+1}) = 1/6[s''(X_i) + s''(X) + s''(X_{i+1})] \tag{5}$$

Substituting equations (2) and (5) into equation (1), the polynomial expression of the natural spline is obtained as follows:

$$\begin{aligned} s(X) = {} & s(X_i) + (X-X_i)[s'(X_i)+1/2(X-X_i).s''(X_i) \\ & + 1/6(X-X_i)^2 .s''(X_i,X_{i+1})] \\ & + 1/6(X-X_i)(X-X_{i+1})[s''(X_i)+s''(X)+s''(X_{i+1})] \end{aligned} \tag{6}$$

In order to obtain the slope at any point between (X_i,X_{i+1}), the equation (1) is differentiated with the following result:

$$\begin{aligned} s'(X) = {} & s(X_i,X_{i+1}) + (2X-X_i-X_{i+1}).s(X,X_i,X_{i+1}) \\ & + 1/6(X-X_i)(X-X_{i+1}).s''(X_i,X_{i+1}) \end{aligned} \tag{7}$$

Equations (4) and (7) allow the evaluation of $s''(X)$ and $s'(X)$

respectively. Using these two equations, equation (6), and the given knots (Xi,Yi), the spline profile can be obtained by plotting s(x) versus X for the range $a < X < b$ where $a = X1$ and $b = Xn$. However, in order to obtain the values of s"(X) using equation (4), s"(X1) and s"(Xn) are arbitrarily chosen as zeros. This will be evidenced in this work by the plots of the profiles. It should be noted here that the function g(X) has been defined as the collection of the natural splines s(X) with no regard to the manufacturability of the profile where tolerances are provided. This works well for drafting purposes. The techniques used to obtain a smooth transition from one section of the geometry to the next have led to other splining methods such as the Biezer and Hermite [11-14] for design purposes.

2.2 The Manufacturing Data

Two basic steps are involved in the manufacturing of a spline geometry such as the turbine blade profile. First, the digitized design data must be splined by a manufacturing splining method to obtain a curve which represents the "ideal" part surface. Secondly, a set of points to be used to define the tool path in a CNC program must be obtained from this curve. These points, which shall be referred to as the manufacturing data, are selected by means of an interpolation procedure based on a defined tolerance. The interpolation process obtains the number of points and their locations such that the defined tolerance is not exceeded. Three examples are provided here to illustrate how the number of points are determined within the interpolation system.

Consider an inside tolerance as shown in Figure 2. In this system, the ideal surface is approximated by chords such as AB, BC, and CD. The number of chords must be obtained before interpolation within each segment. Figure 3 shows a system with tangential tolerance and Figure 4 shows a bilateral tolerance. The following equations can be shown to apply to each of the systems.

2.2.1 Chordal(Inside) Tolerance.

The tolerance, $t = (1/c).(1 - \cos\varphi/2)$ (8)

where c = the curvature of the segment

φ = the incremental angle between adjacent points

The number of chords, n, required for the specified t value is then given as $n = \varphi/\theta$, where θ is the included angle of the segment.

That is $n = \theta / 2 \cos^{-1}(1-ct)$ (9)

and $\varphi = 2 \cos^{-1}(1-ct)$ (10)

Thus knowing the values of φ and n, the values of X and the Y coordinates for subsequent points are then obtained [15]. For CNC programming, the diameter of the tool must be taken into consideration. It can be shown that for a tool radius of r, the value of φ is given as:

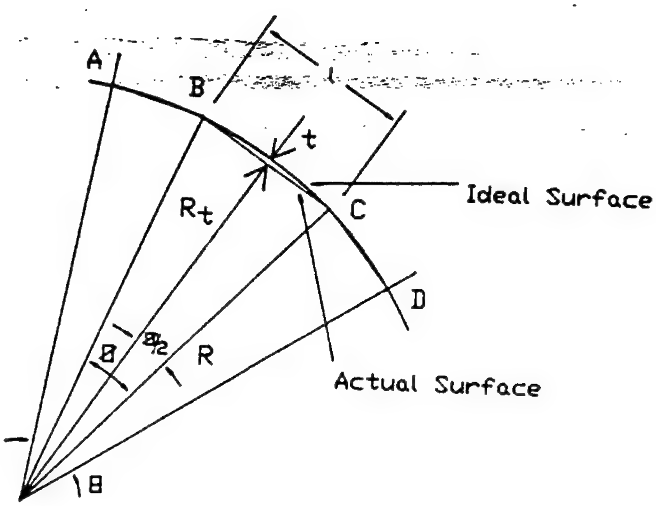
$$\varphi = 2 \cos^{-1}[1 - (ct/1-r)] \quad (11)$$

Also the length of each chord is given by:

$$L = (2/c).\sin \varphi/2 \quad (12)$$

2.2.2 Tangential(Outside) Tolerance.

$$\varphi = 2 \sec^{-1}(1 + ct) \quad (13)$$



R = Radius of Circular Segment

θ = Included angle of Segment

β = Included angle of each chord which leads to the allowable t

Fig. 2 : Definition of Chordal (Inside) Tolerance

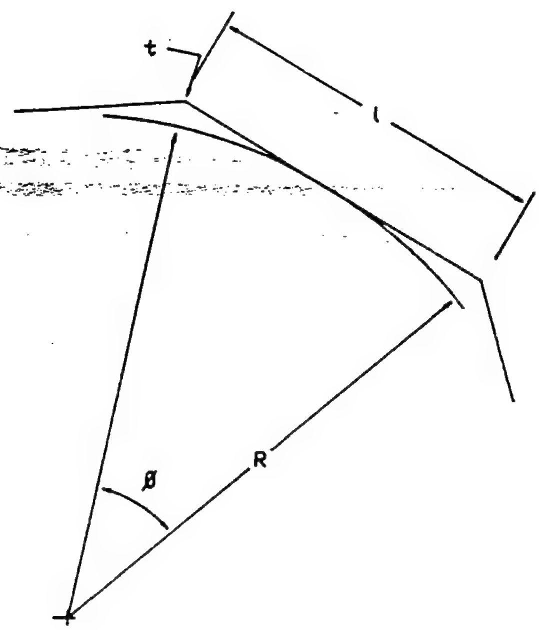


Fig. 3 : Definition of Tangential (outside) Tolerance

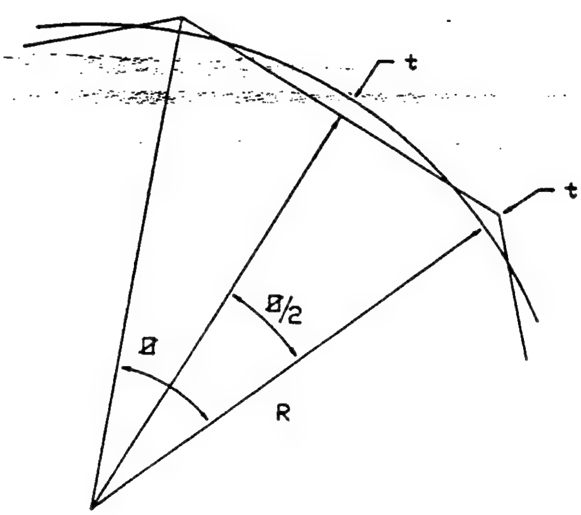


Fig. 4 : Definition of Bilateral Tolerance

For a tool diameter of 2r,

$$\varphi = 2 \sec^{-1}[1 + ct/(1-r)] \tag{14}$$

$$n = \varphi/2 \sec^{-1}[1 + ct/(1-r)] \tag{15}$$

$$L = 2/c \tan \varphi/2 \tag{16}$$

2.2.3 Bilateral Tolerance.

$$\varphi = 2 \cos^{-1}[(1-ct)/(1+ct)] \tag{17}$$

or $$\varphi = 2 \cos^{-1}[(1-ct-cr)/(1+ct-cr)] \tag{18}$$

for a tool radius, r.

and $$L = 2[(1+ct)/c] \sin \varphi/2 \tag{19}$$

The bilateral tolerance method was used in this work since it is the most commonly used one in CAM systems. From the above equations, it can be seen that the correctness of the manufacturing data depends on the spline curve as well as the specified tolerance. The surface or profile produced consists of a series of straightline cuts which approximates the required one. The Tabulated Cylinder (TABCYL) in APT is an example of a splining technique which takes the specified tolerance into account to generate the coordinate points for the tool path [16].

CHAPTER III
GENERAL PROCEDURE

Design data for a typical twisted turbine blade was obtained from the Westinghouse Electric Corporation's turbine components manufacturing plant in Winston-Salem, North Carolina. The data consisted of a set of coordinates at sections of the blade ranging from the tip to the root. Each set of data for a section, was made up of 12 coordinates on the convex side and 11 on the concave side. The basic hardware needed for the data derivation and analysis were:

- IBM PC-AT with FORTRAN 77 Compiler.
- NEC PC with Graphics Terminal.
- CAM Software to generate NC Programs.Bridgeport's EZ-CAM System was used.
- Drafting Software (AUTOCAD)
- Plotter (HP Plotter)

The general procedure used in this work is as follows:

1. An interpolating cubic spline program as described above was used to fit a spline through the control points of both the convex and the concave side of the turbine section. With this program, the curve parameters y, y', y" at 100 matching points were obtained and used to plot x versus these parameters using the AUTOCAD drafting software. These plots represent the design parameters and profile. This step was performed using the IBM PC-AT.

2. To obtain the manufacturing data, the EZ-CAM milling and Bridgeport Operating System Software (BOSS) were used. The Simplified Interactive Part Program (SIPP) of the EZ-CAM was used to define the knots and the tool path. Within the EZ-CAM system is the TABCYL splining routine which outputs the coordinate points for a given tolerance value. The TABCYL routine selects

the tool path points in such a manner that the maximum deviation of the tool path coordinates is not more than the defined tolerance. The tool moves linearly between the defined points to approximate the profile. The data obtained was post-processed by means of the BOSS 8M post-processor so that it may be machined on the Bridgeport Series 1 R2E3 Milling machine. This was done for several sections of the turbine blade under investigation.

3. The CNC part program was then downloaded to the IBM PC-AT. To achieve this, EZ-COM, a communication utility in the EZ-CAM software and XTALK, a communication package for the IBM PC were used. The purpose of this was to input the manufacturing data to the splining software for analysis.

4. In order to analyse the manufacturing data, the coordinates generated by the EZ-CAM system were derived from the NC program sent to the IBM-PC. A special software was written to obtain only the X and Y values of the NC progam. Other information not required for this analysis was left out by the program.

5. Next, the manufactured curve parameters (y, y') were obtained at 100 matching points. The values of y" are undefined here since the paths followed by the tool are straightline segments. The y-coordinate was calculated by simple linear interpolation and the y'(slopes) at the matching points were the slopes of the linear segments within which the matching point lies. Data was generated to plot x versus y and y' for the profile by means of AUTOCAD. These plots were then superimposed on the design ones for comparison. Figures 5 through 16 show examples of these plots. The values of the other parameters of the profiles are provided in the Appendix.

6. The geometric properties of the design and the manufactured profile were next calculated. Here, finite element methods were employed. Each region under consideration was divided into narrow rea, centroids, and the moments of inertia calculated. The total properties were then obtained from those of the elemental strips.

CHAPTER IV

COMPARISON OF DESIGN AND MANUFACTURING PROFILES

The above procedure was used to obtain the plots of the profiles, and slopes for three sections of the turbine blade. Figures 5 through 16 show the variation of the manufacturing characteristics from the design ones. The profiles may be compared using Figures 5, 8, 11, and 14. The differences between these figures are due to the specified tolerances. The tolerances are specified at 0.1 inch for Figure 5, 0.01 inch for Figure 8, 0.001 inch for Figure 11, and at 0.0001 inch for Figure 14. It can be seen that as the tolerance gets tighter, the deviation of the profile at the trailing edge gets larger. This can be explained by the fact that the design profile approaches a straightline at this section (AB) while the manufacturing profile is curved more by the method used for the interpolation. The design curve has only two points between A and B while the number of segments, and thus points, between these two points become larger and larger as the tolerance value decreases. This can be explained by Equations (17) and (18). Since the interpolation process takes place between two consecutive points, and the boundary conditions must be satisfied for each interpolation, the deviation of the manufactured profile tends to get worse as the tolerance gets tighter.

The slopes at various points on the convex side may be compared for the various tolerances by referring to Figures 6, 9, 12 and 15. The slopes show the same results as the profile. The mismatch of the profiles, especially for section AB can be clearly seen from the slopes. Figures 7, 10, 13, and 16 show the variations on the concave side of the section. The concave sides seem to match up very well and were therefore not pursued any further.

The profiles and parameters shown in Figures 5 through 16 are for section of the blade taken near the tip. In order to check the behavior of the curve for other sections two other sections

were taken. Figures 17 and 18 show the profile and the plot of the slopes respectively, for a section in the middle of the blade. Figures 19 and 20 also show the profile and slopes respectively, near the root. It can be seen that the section becomes more curved as it gets nearer to the root. It can also be seen that as the curvature increases at section AB the profiles match better. This is due to the fact that the straightline section of the design profile is replaced with a curved one which is subject more to splining, the same way the manufacturing system performs.

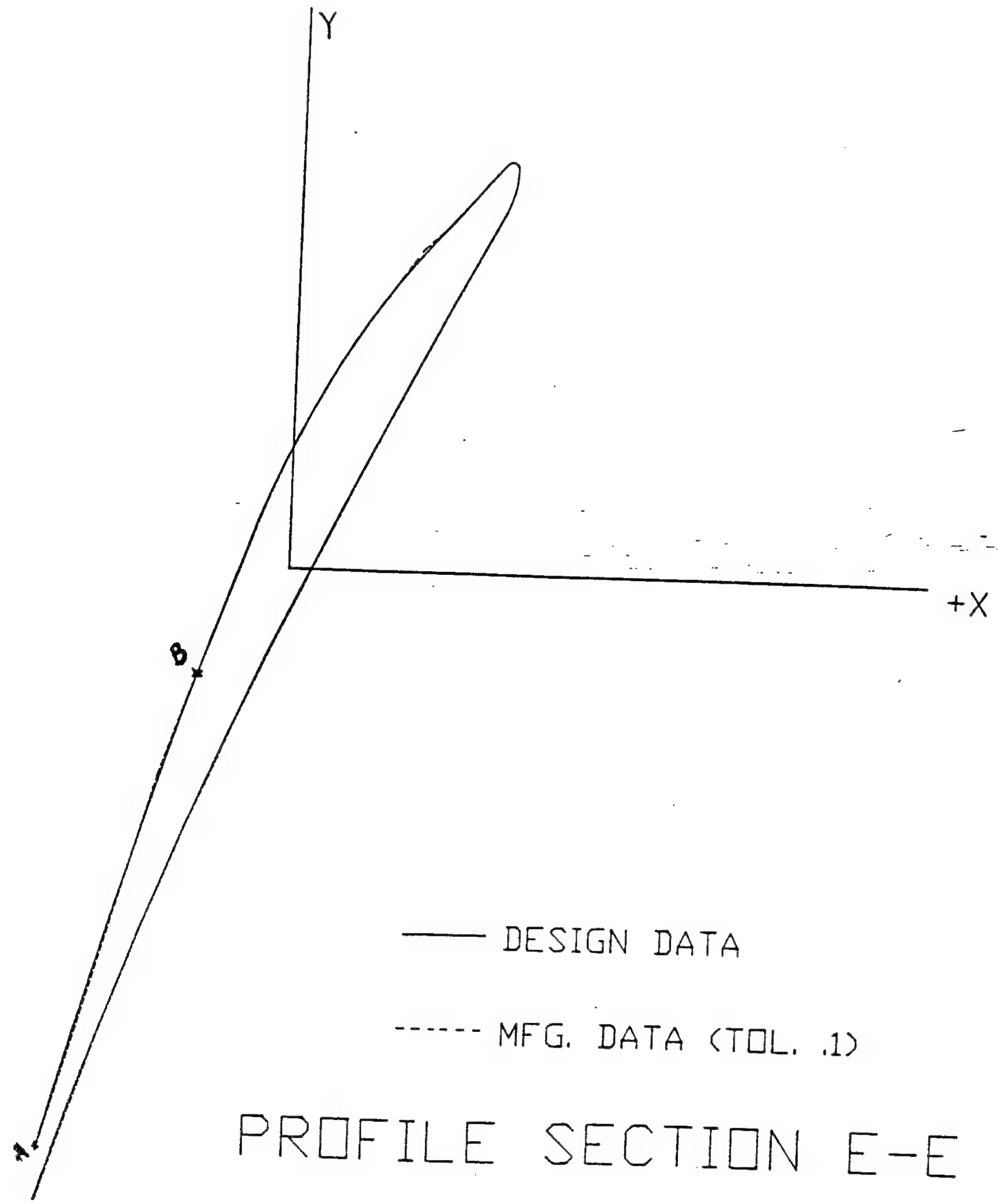


Figure 5 Superposition of Design and Manufacturing Profiles Section E-E (Tol=0.1 in.)

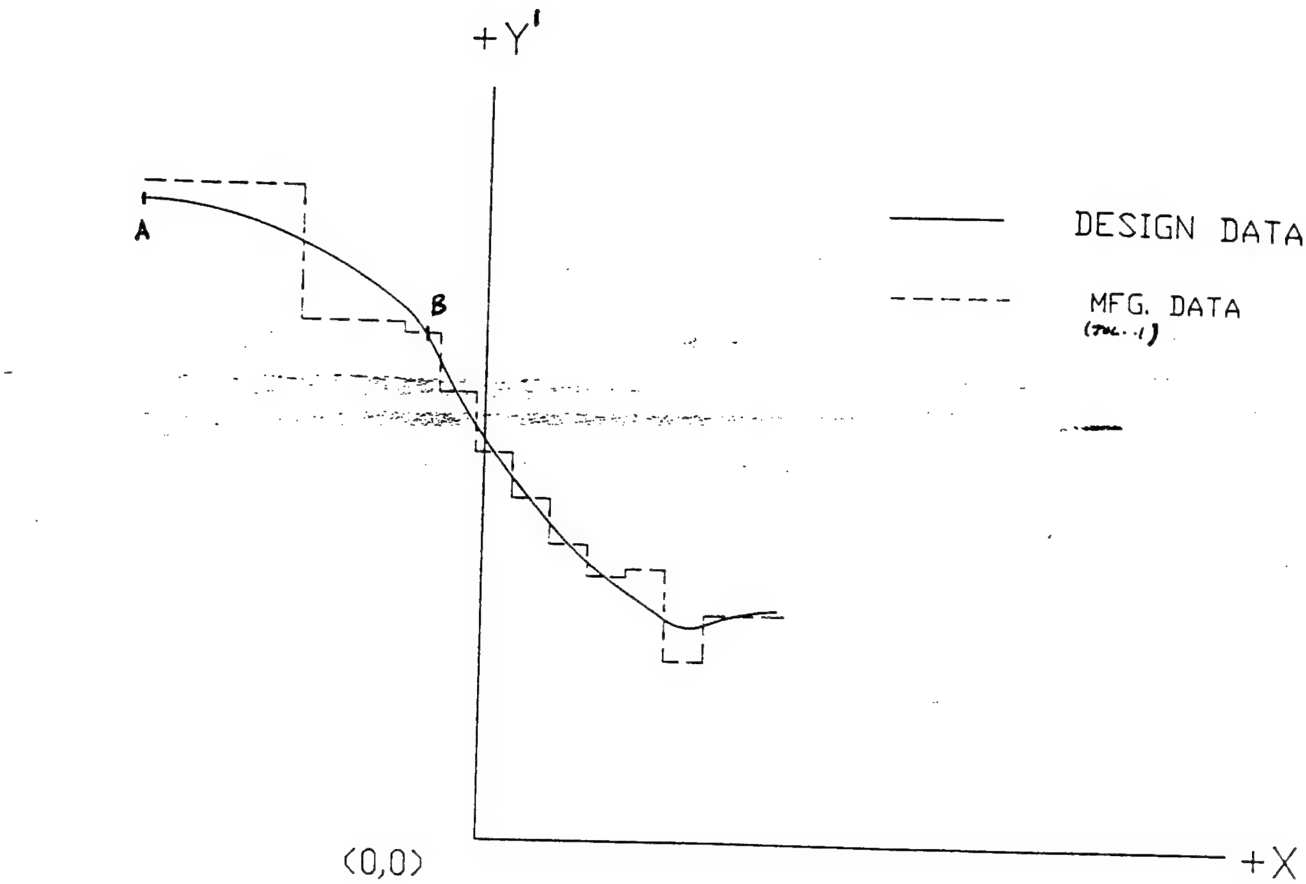


Figure 6 Superposition of Design and Manufacturing Slopes Section E-E, Convex Surface (Tol.=0.1 in.)

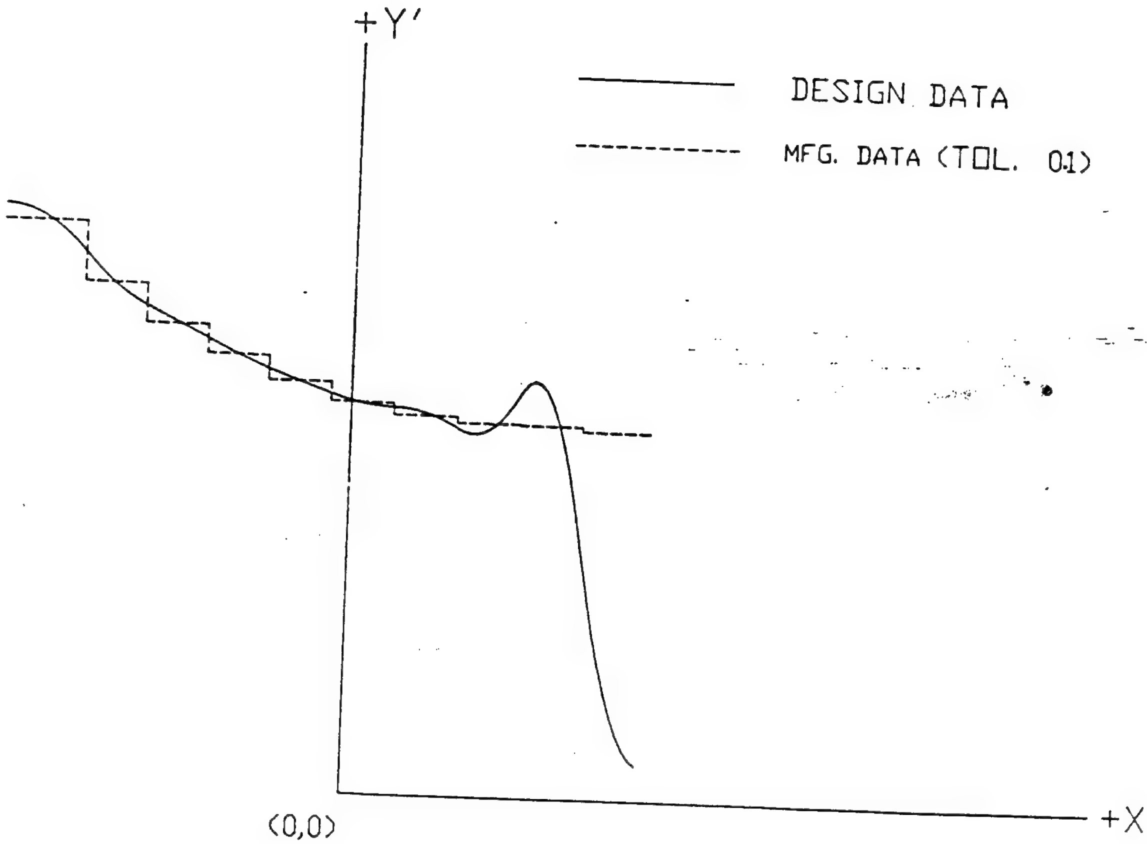


Figure 7 Superposition of Design and Manufacturing Slopes
Section E-E, Concave Section (Tol.=0.1 in.)

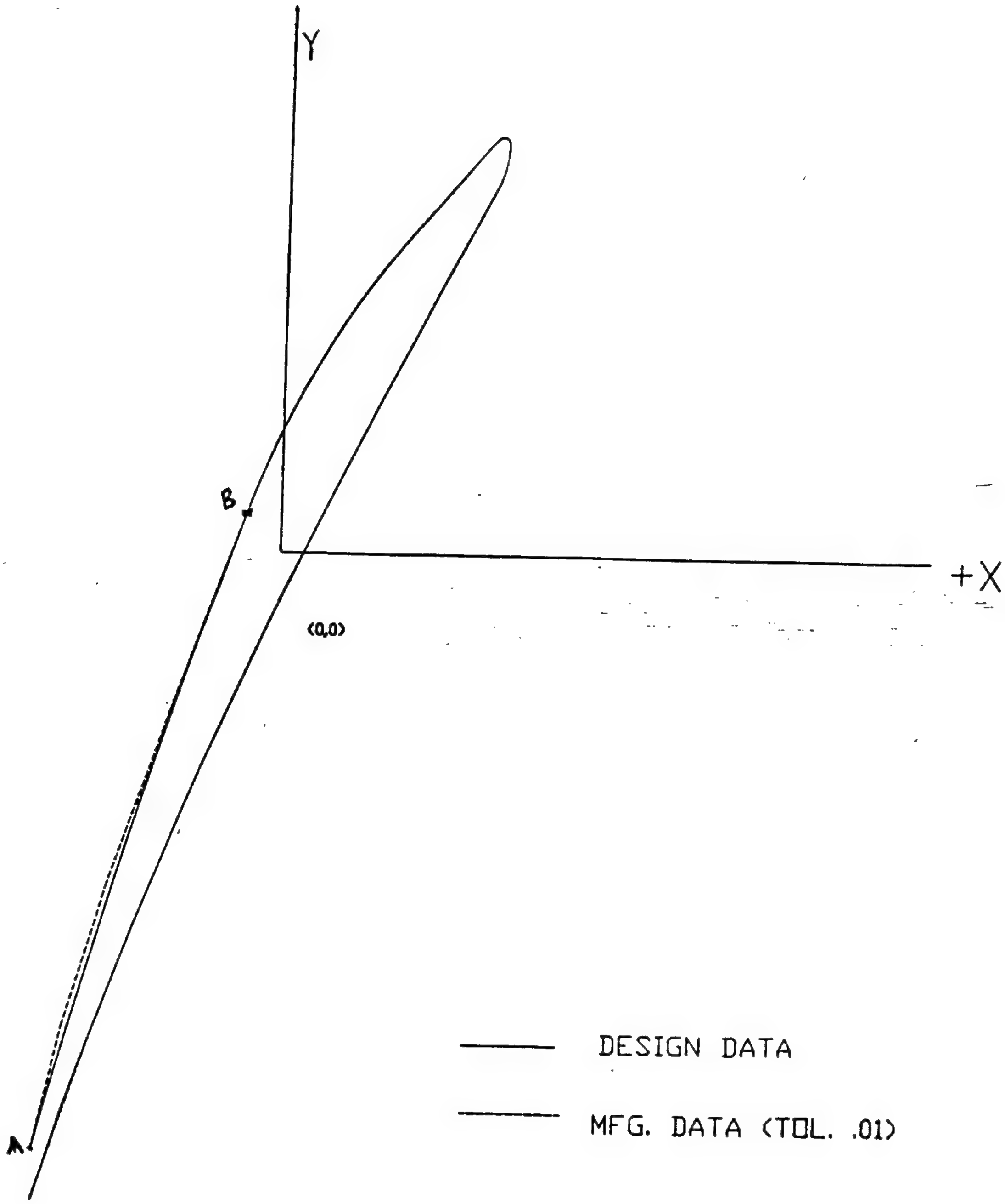


Figure 8 Superposition of Design and Manufacturing Profiles Section E-E, (Tol.=0.01 in.)

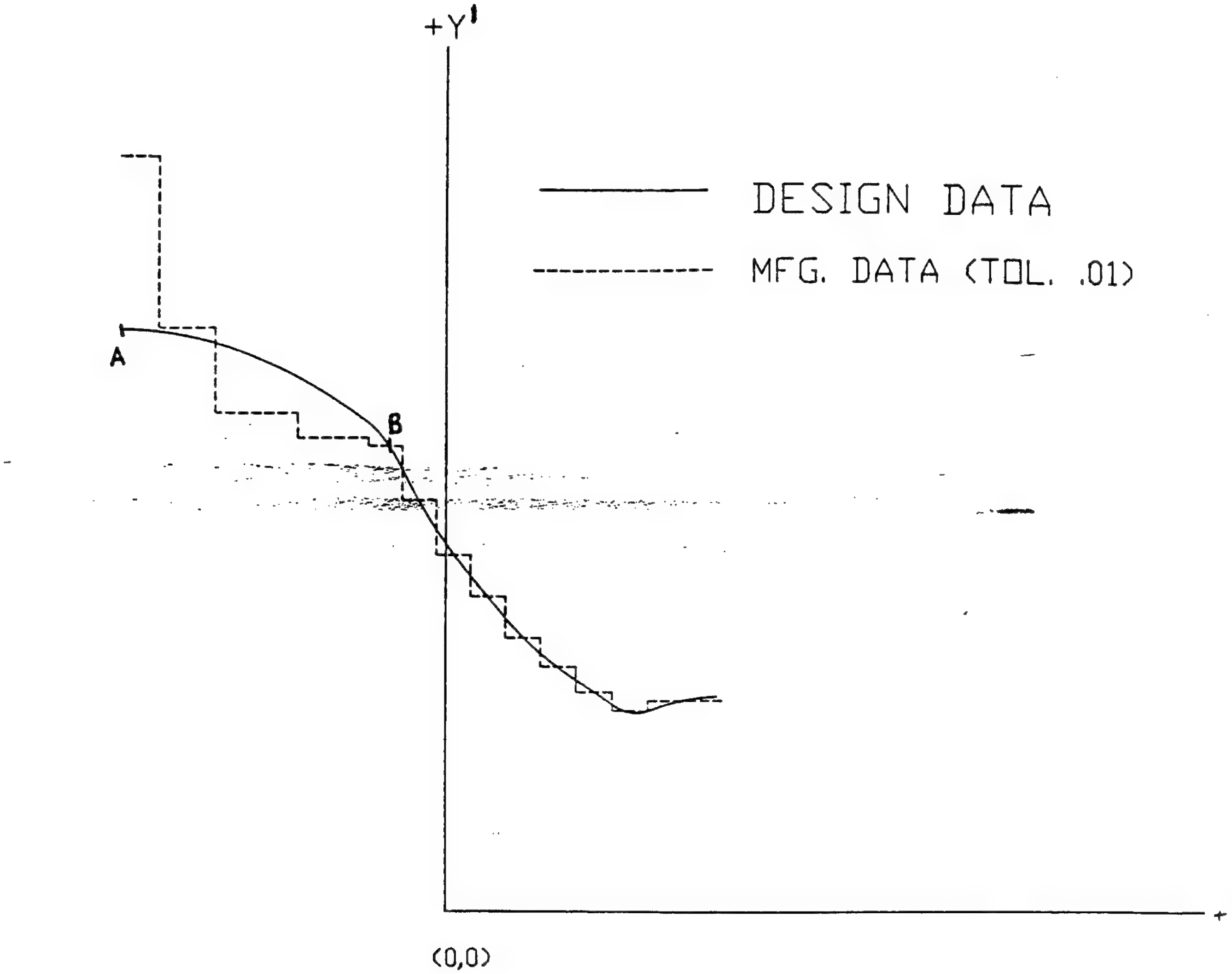


Figure 9 Superposition of Design and Manufacturing Slopes
Section E-E, Convex Surface (Tol.=0.01 in.)

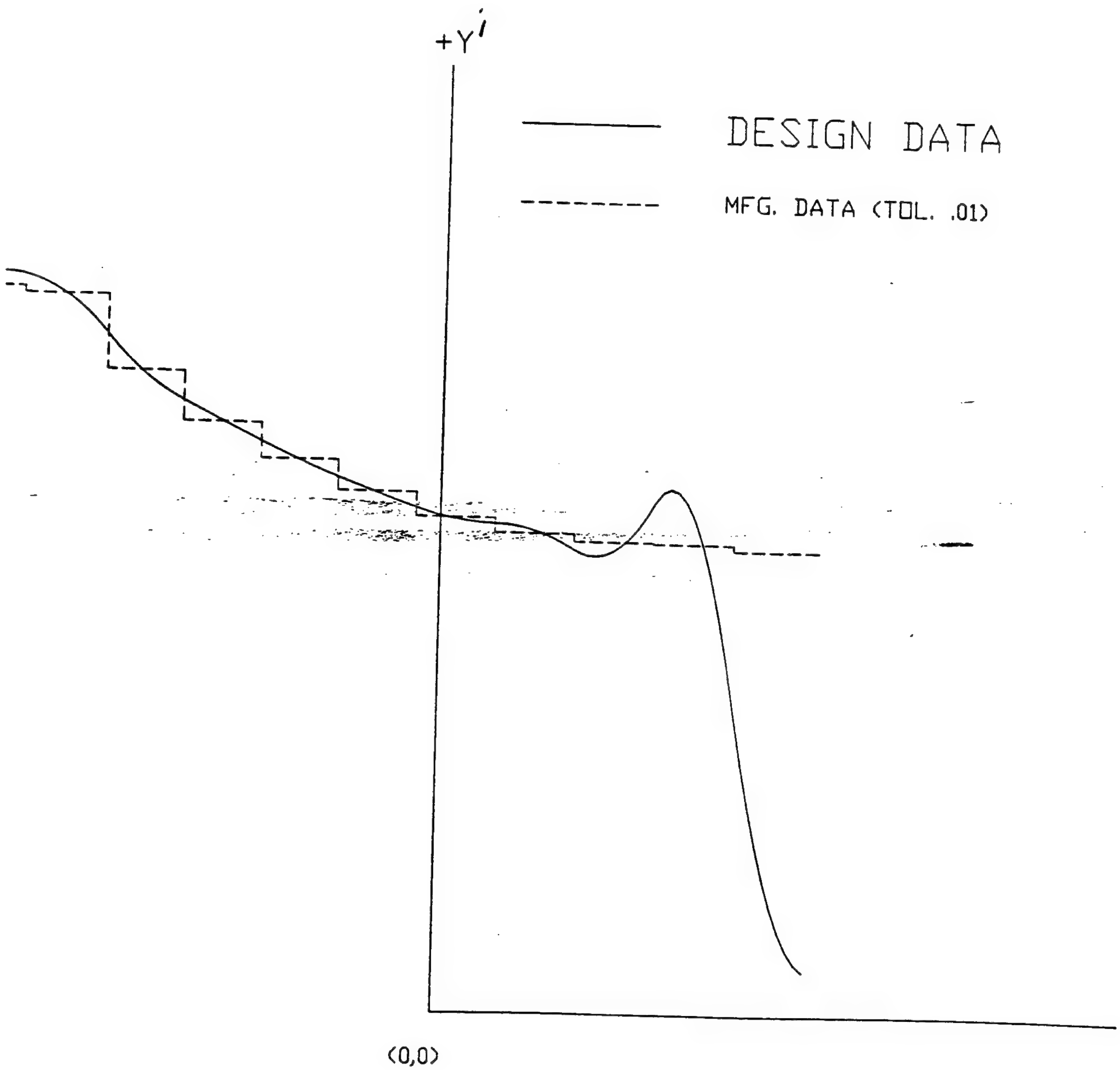


Figure 10 Superposition of Design and Manufacturing Slopes Section E-E, Concave Surface (Tol.=0.01 in.)

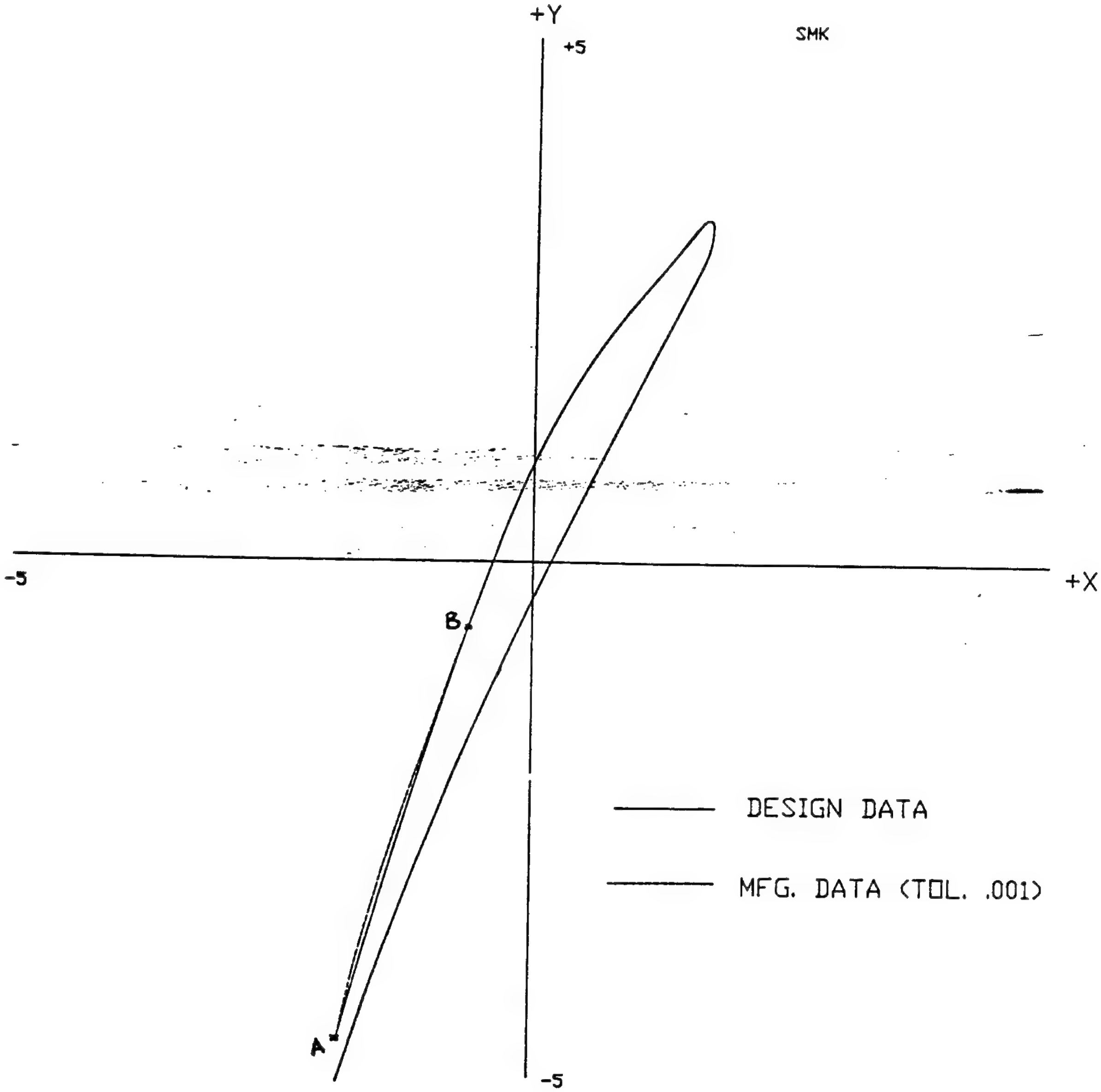


Figure 11 Superposition of Design and Manufacturing Profiles
Section E-E (Tol.=0.001 in.)

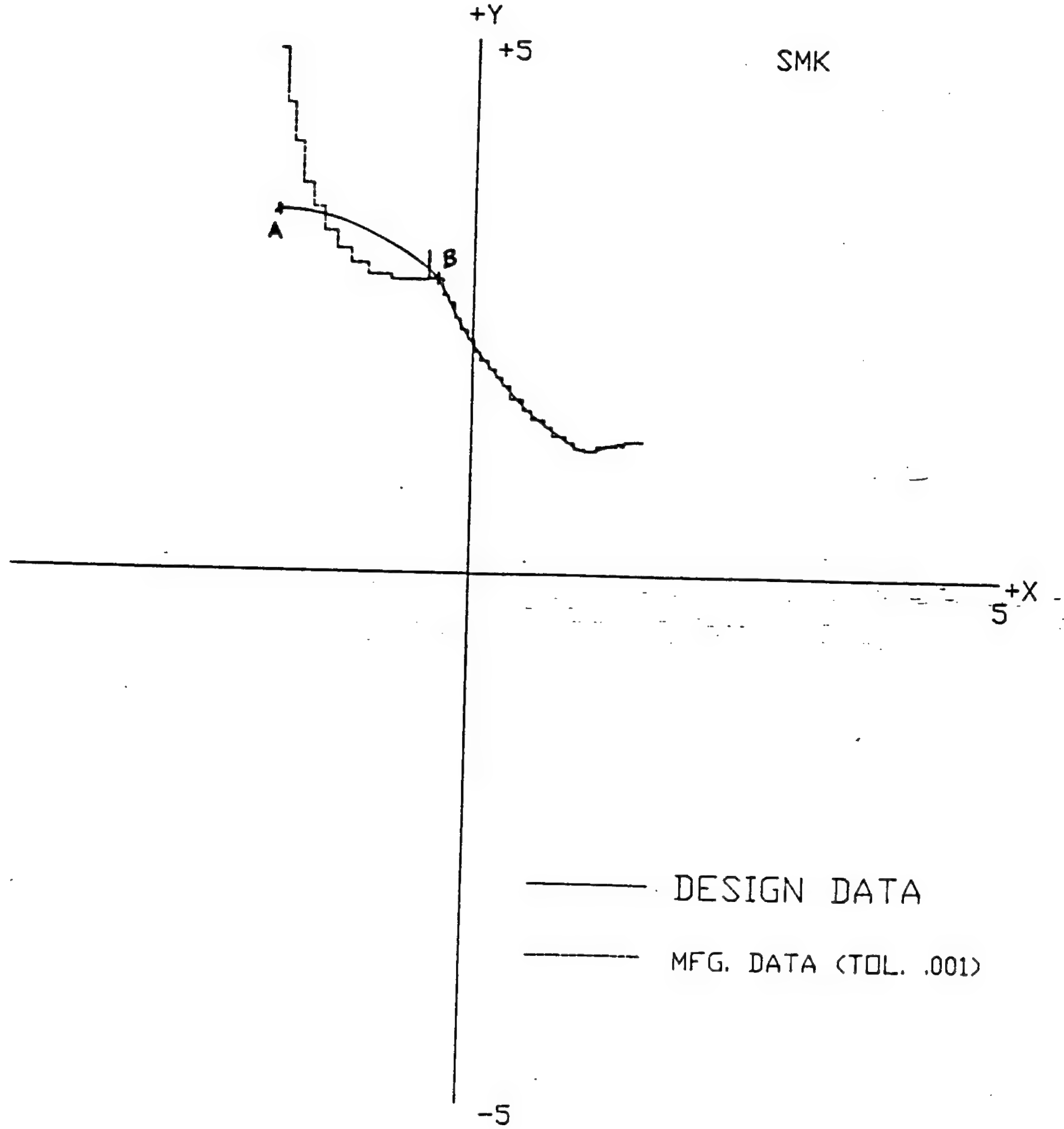


Figure 12 Superposition of Design and Manufacturing Slopes Section E-E, Convex Surface (Tol.=0.001 in)

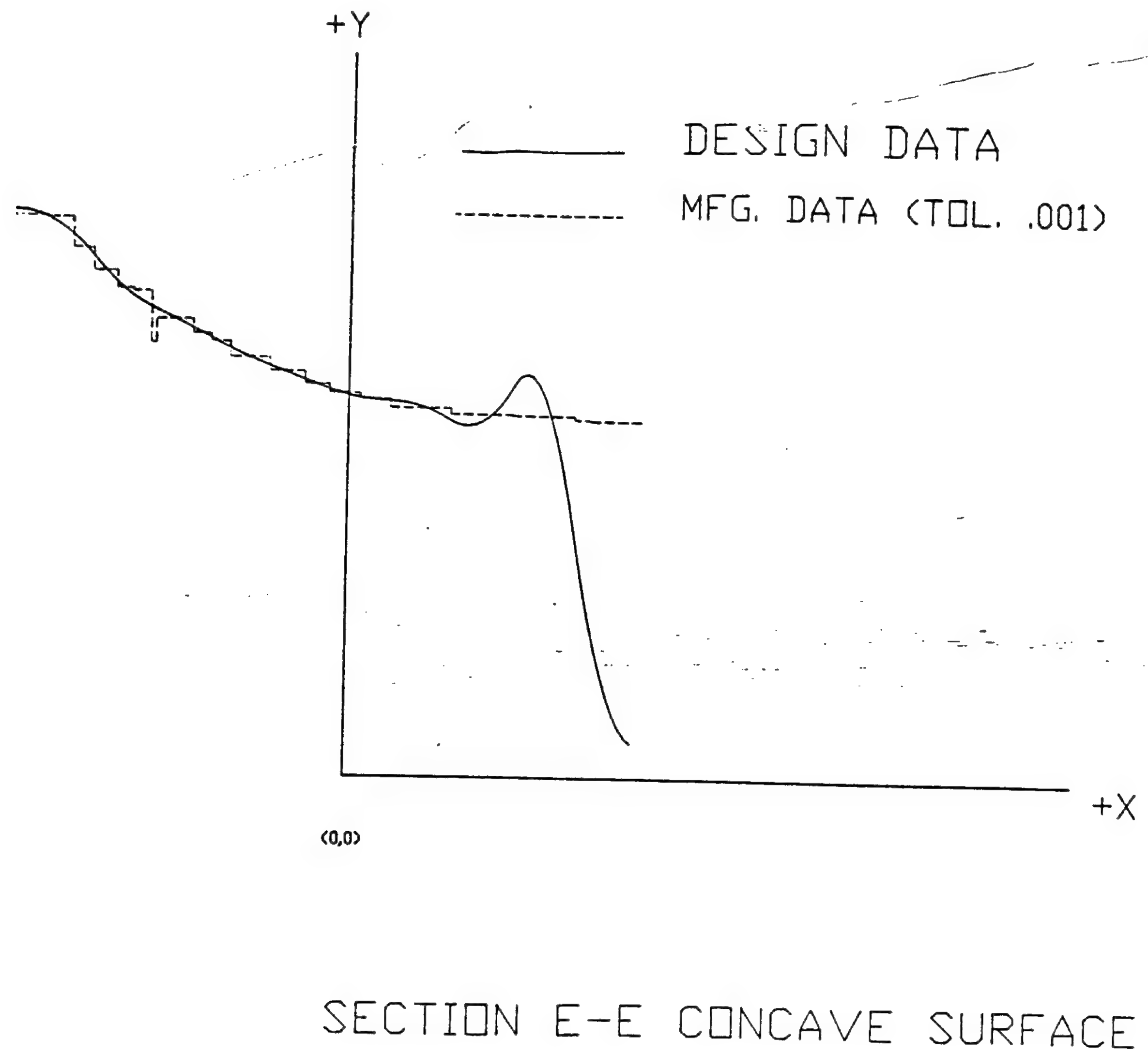
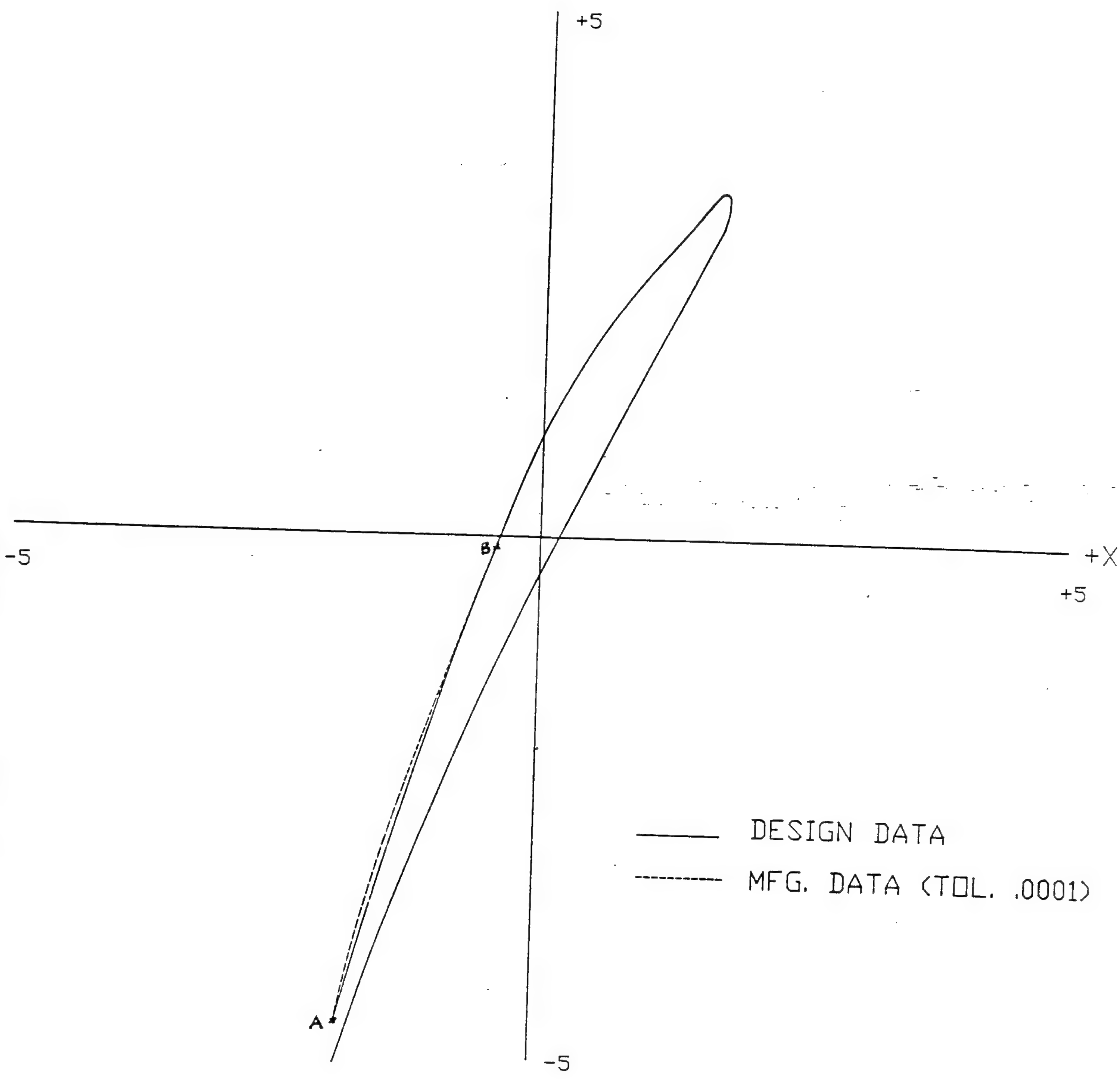


Figure 13 Superposition of Design and Manufacturing Slopes Section E-E, Concave Surface (Tol=0.001 in.)



PROFILE SECTION E-E

Figure 14 Superposition of Design and Manufacturing Profiles Section E-E (Tol=0.0001 in.)

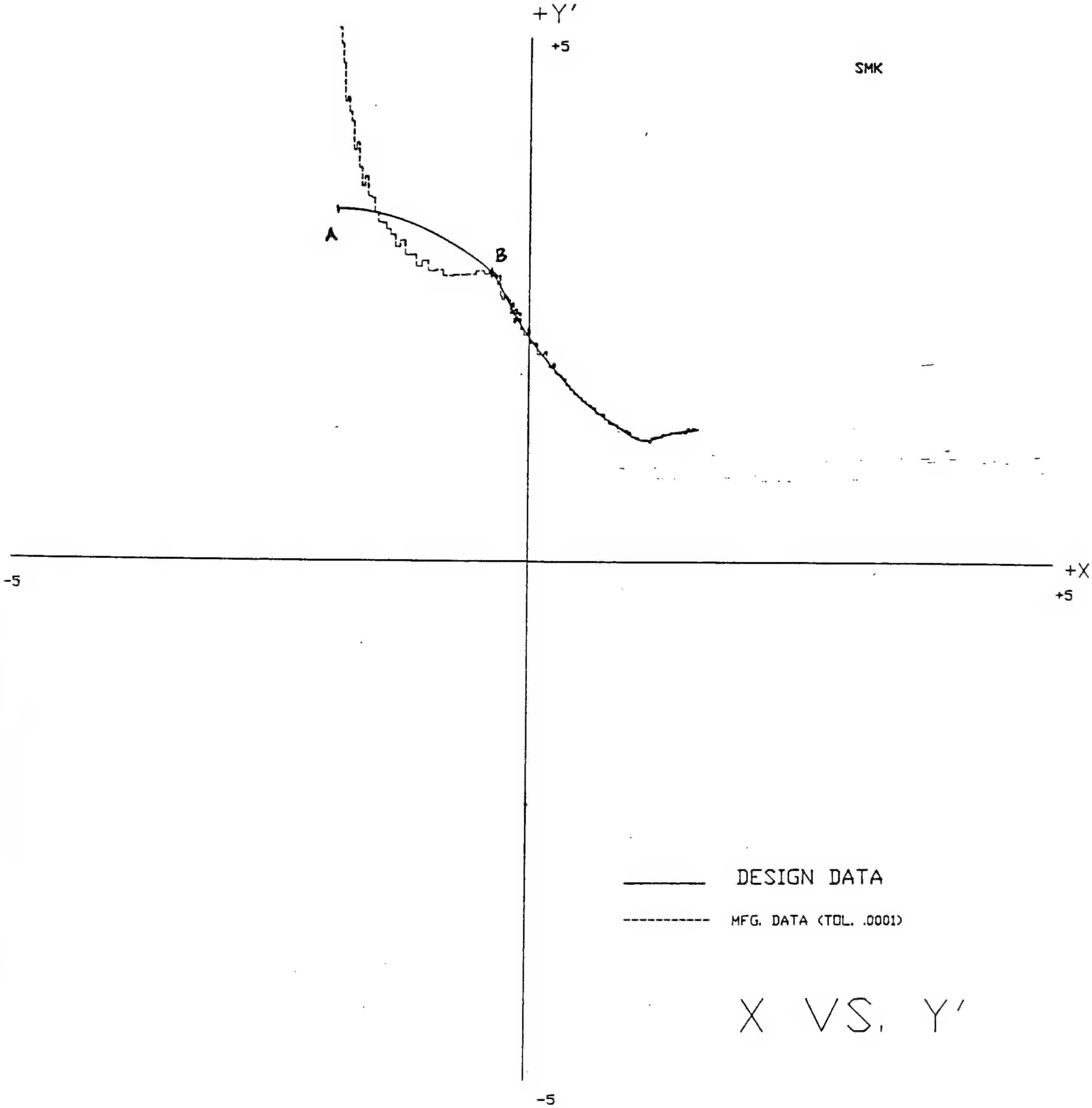


Figure 15 Superposition of Design and Manufacturing Slopes Section E-E, Convex Surface (Tol.=0.0001 in.)

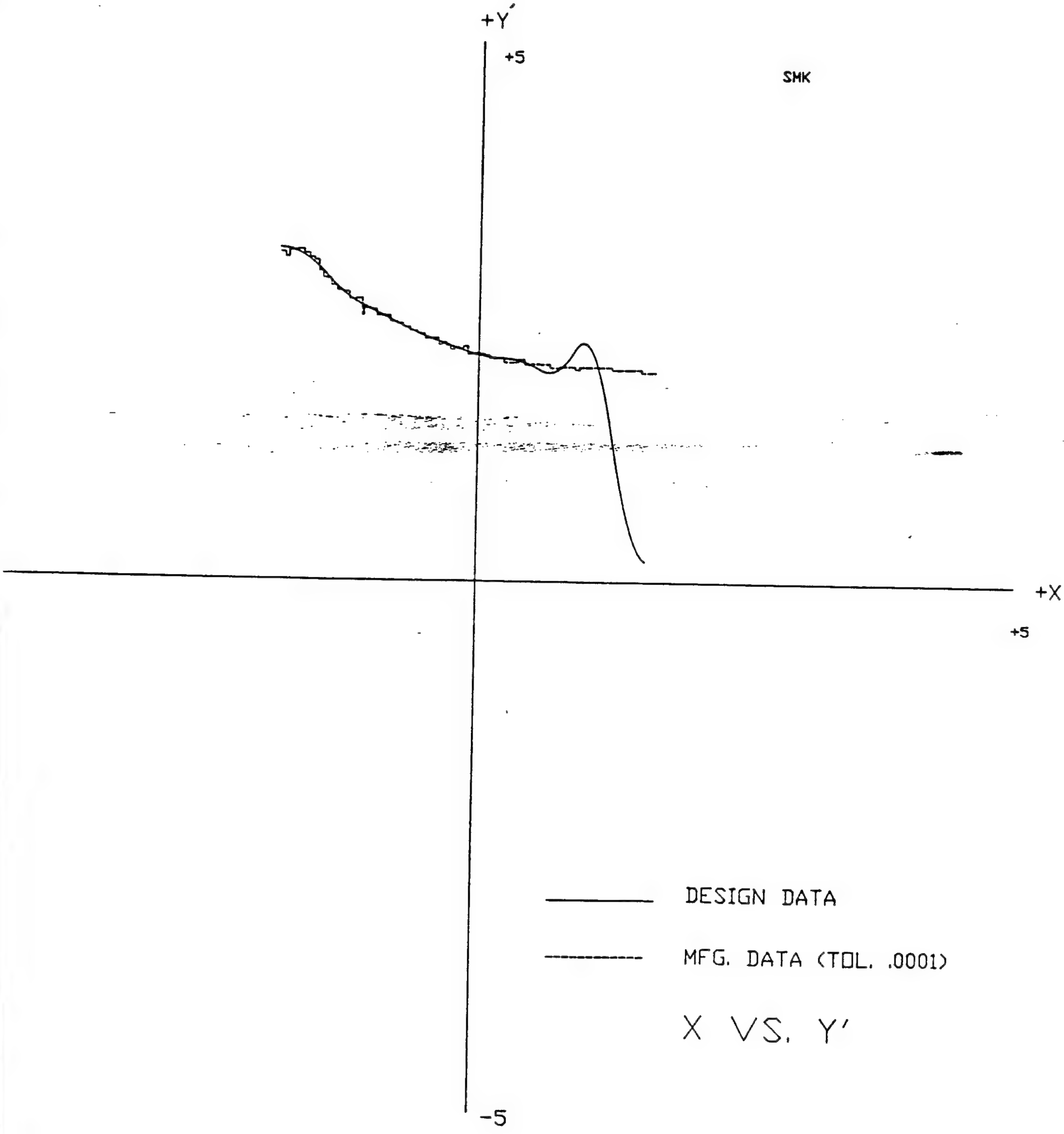


Figure 16 Superposition of Design and Manufacturing Slopes Section E-E, Concave Surface (Tol.=0.0001 in.)

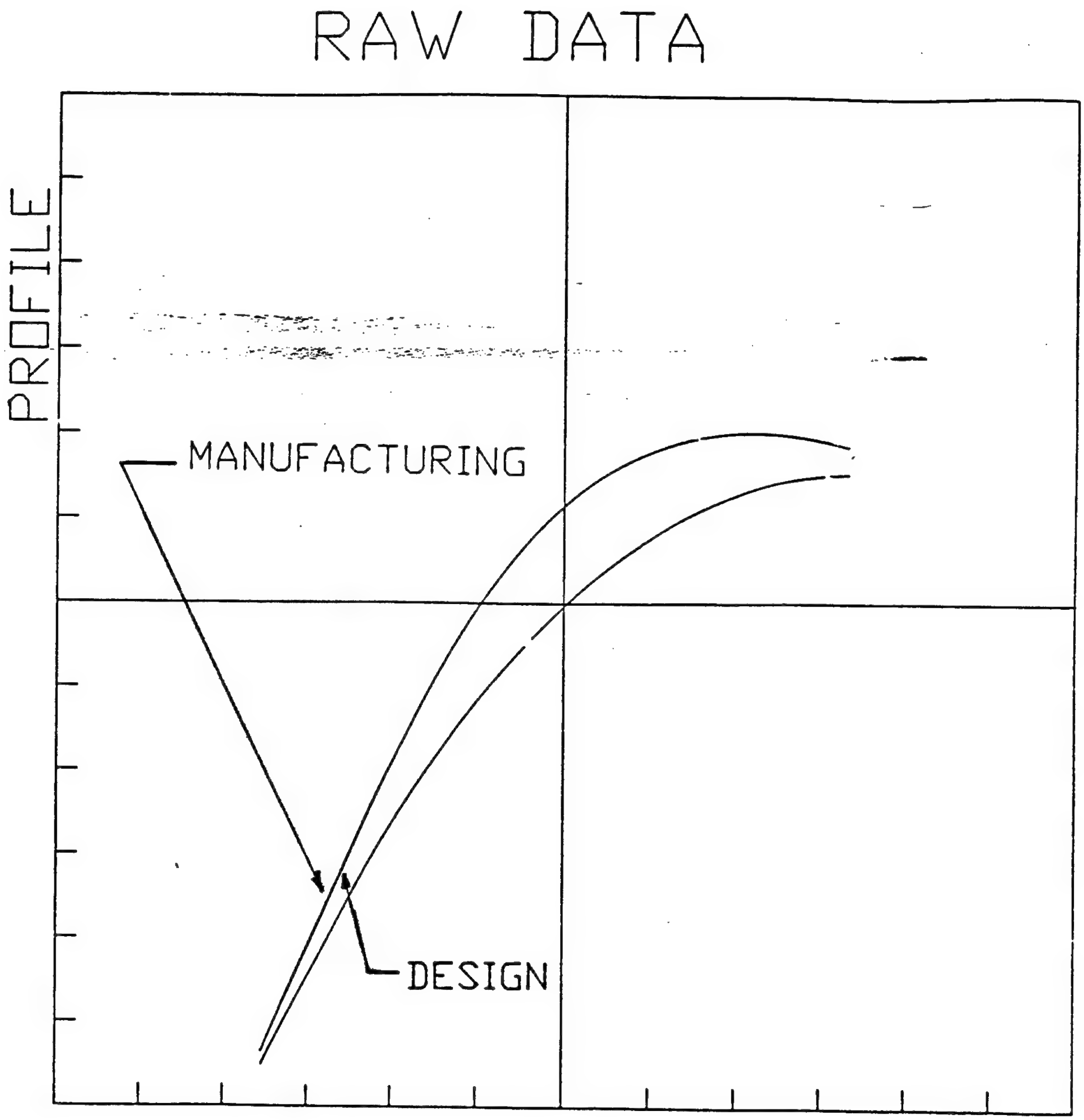


Figure 17   Superposition of Design and Manufacturing Profiles
Section K-K (Tol=0.0001 in.)

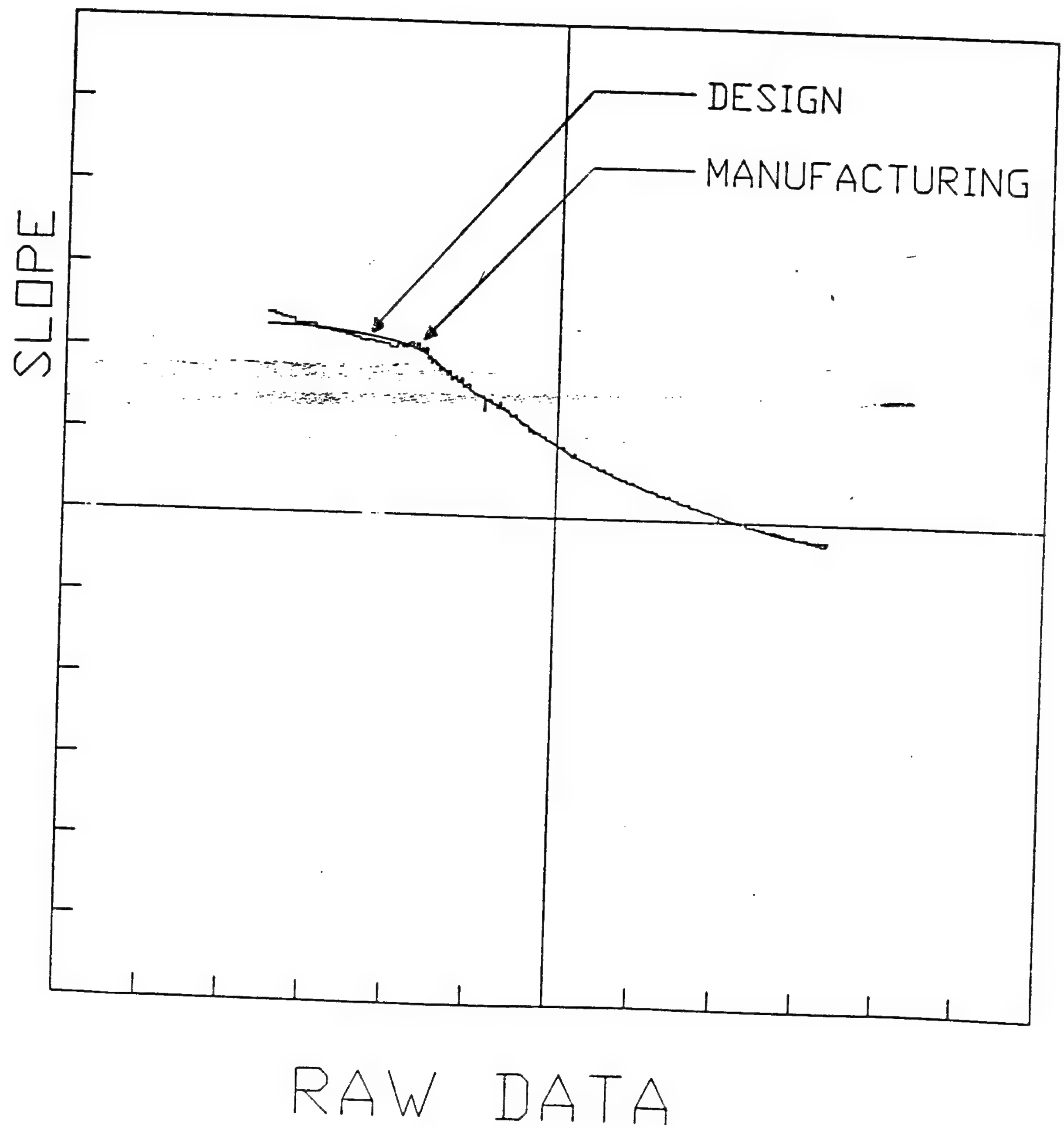


Figure 18 Superposition of Design and Manufacturing Slopes Section K-K, Convex Surface (Tol=0.0001 in.)

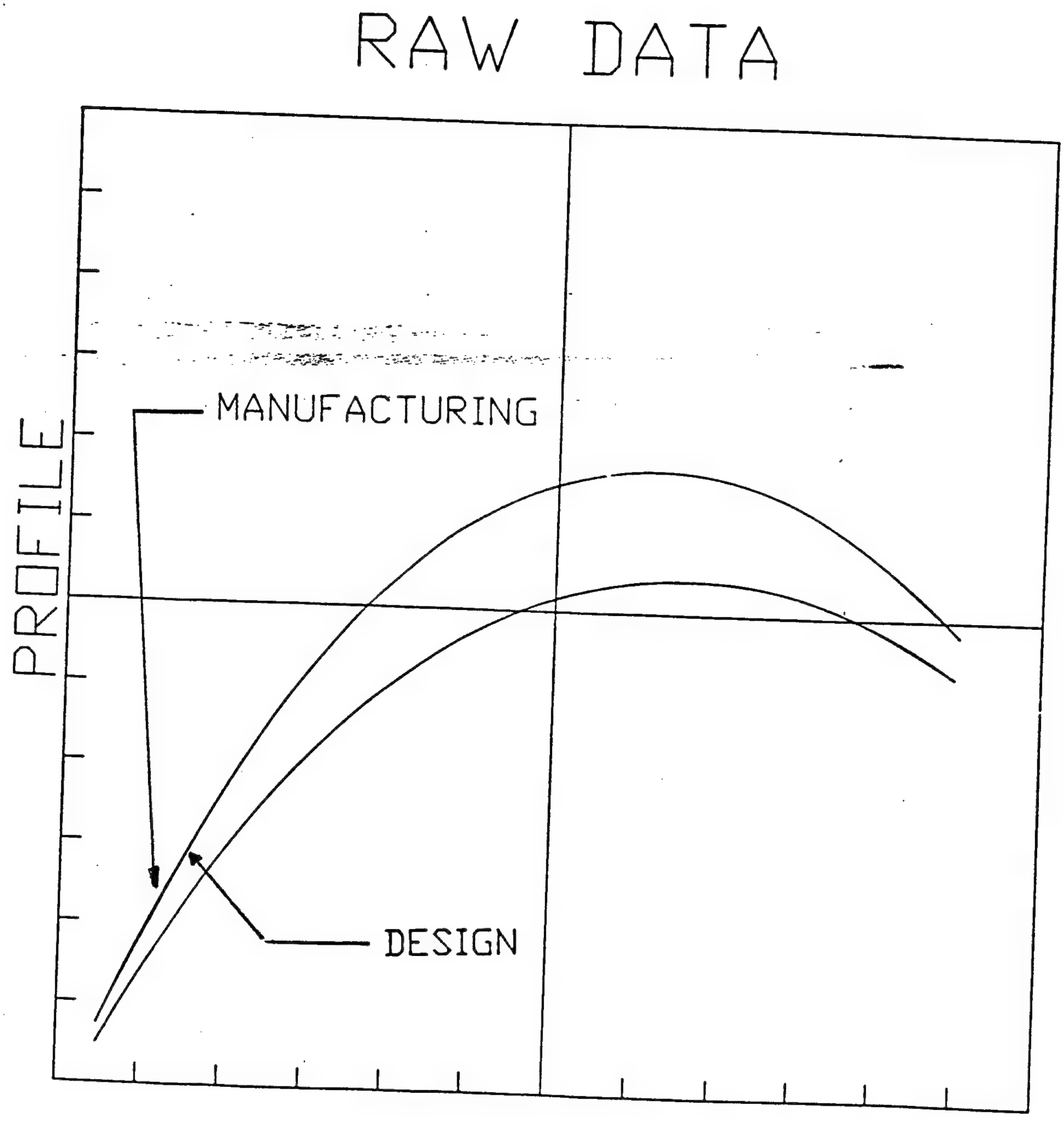


Figure 19 Superposition of Design and Manufacturing Profiles Section R-R (Tol.=0.0001 in.)

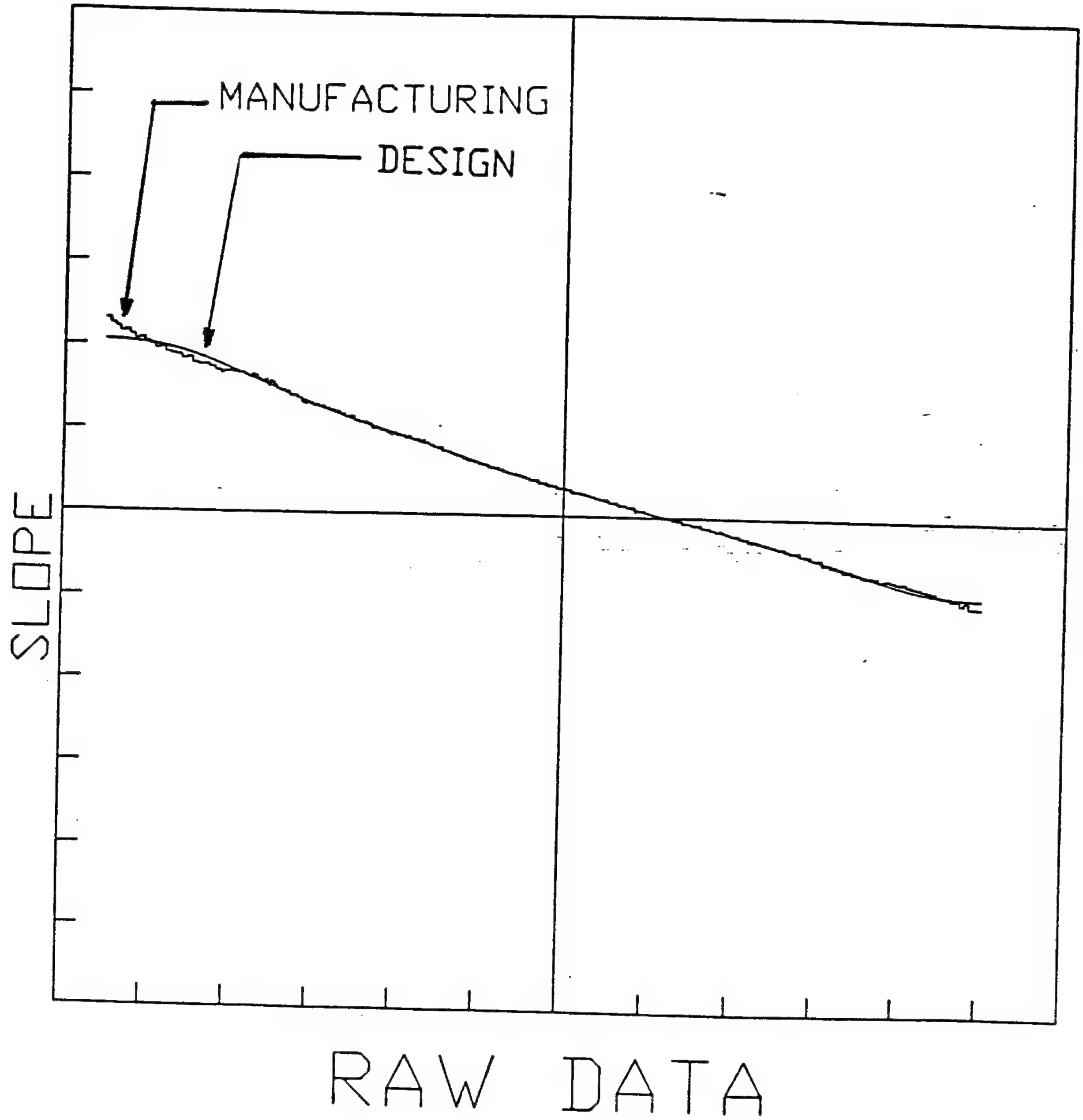


Figure 20 Superposition of Design and Manufacturing Slopes Section R-R, Convex Surface (Tol.=0.0001 in)

CHAPTER V

DESIGN AND MANUFACTURING DATA INTEGRATION

The above discussions indicate that there is the need to take a look at the manner in which points are defined for the manufacturing system from the design profile. The number of points provided as well as their locations are of the utmost importance. For example, even though the portion AB of the sections nearer to the trailing edge may approach a straightline, the splining technique does not recognize this and the smoothing process, which blends the various natural splines together, causes the profile to deviate from the design one.

This section of the report provides a method for additional point selection, the purpose of which is to improve the manufacturability of the turbine blade profile. This investigation was performed in two stages. The first stage consisted of the visual selection of extra points at locations where the slopes and profile deviations were largest when the two profiles are superimposed on each other. After several trials, it was determined that this matching process could be done systematically. At the second stage, this method was converted into an algorithmic procedure and a program was written in FORTRAN 77 on an IBM PC-AT to go through an iterative process to select the additional points needed for the two profiles to match. This algorithm, which is discussed below, is different from simply reconstructing the spline by sampling [17].

The objective of this algorithm is to improve the manufacturability of the turbine blades by the proper selection of the knot points. It will be demonstrated that by the proper selection of the control points from the designed profile, a better match between the two profiles can be achieved by milling alone. Thus additional finishing processes may be eliminated to reduce the

manufacturing time and cost. It is also believed that if the manufacturing capabilities of the system at hand are considered while designing the profile, a more producible part may result. This amounts to designing the profile with its manufacturability as one of the design constraints. This would result in altering the design to improve manufacturability at the design stage while maintaining design tolerances.

In this algorithm new control points are added, if necessary, to achieve a better match between the design and the manufacturing profiles as well as the slopes at the knots and at any other points of interest. The description of the algorithm is as follows:

a) Using the original (raw) design points, the design profile and the slopes are obtained by the C-spline routine as discussed.

b) Using the raw design points again, and the specified tolerance, the manufacturing spline is obtained with the EZ-CAM system. The slopes are then obtained using a subroutine from the C-spline routine.

c) An appropriate number of matching points are then selected. In this work, 100 matching points are used.

d) The difference between the Y-coordinates of the two profiles at each matching point is calculated.

e) The standard deviation of the differences in the Y-coordinates is calculated.

f) Regions are found where the difference is beyond a specified acceptable limit. The acceptable limits are specified as either $\pm\sigma$, or $\pm 2\sigma$, or $\pm 3\sigma$, where σ is the standard deviation of the differences in the Y-coordinates.

g) For each unacceptable region, the difference between the slopes of the two profiles is calculated. The maximum and minimum slope differences are found.

h) The x-coordinates corresponding to the maximum and the minimum slopes are then evaluated. The mean of these two coordinates represents the x-coordinate of the new additional point.

i) At the x-coordinate of the new point, the y-coordinate on the manufacturing profile is found by simple linear interpolation. Thus the x and y coordinates of the new point are obtained.

j) The new points are added to the knots provided. Steps (a) through (f) are repeated. At step (e), the new standard deviation of the differences of the ordinates is compared with the previous one. If the percent reduction is less than a specified amount (10% in this work), the iteration process is terminated and the other geometric parameters checked.

k) The percent changes in the geometric properties are calculated. The new points are considered acceptable only when the deviations are within acceptable design limits.

Figure 21 shows the procedure for obtaining the new X-coordinate after an unacceptable region has been determined. Figure 22 shows the two profiles after two new coordinates have been added to the original design data. These new points were obtained using this method. A comparison of the profiles can be seen more clearly by means of Figures 23, 24, and 25. In each of these figures, the pair of plots on the far left represents the superposition of the manufacturing profile on the design one when the raw design data is used. The pair of plots in the center, represents the job done by an operator by a trial and error method in which he tries to match the two profiles by assuming the coordinates of the new point. The pair of plots on the far right represents the result of the algorithm developed for this project. The algorithm method, which is systematic, can be seen to agree very well with the manual method.

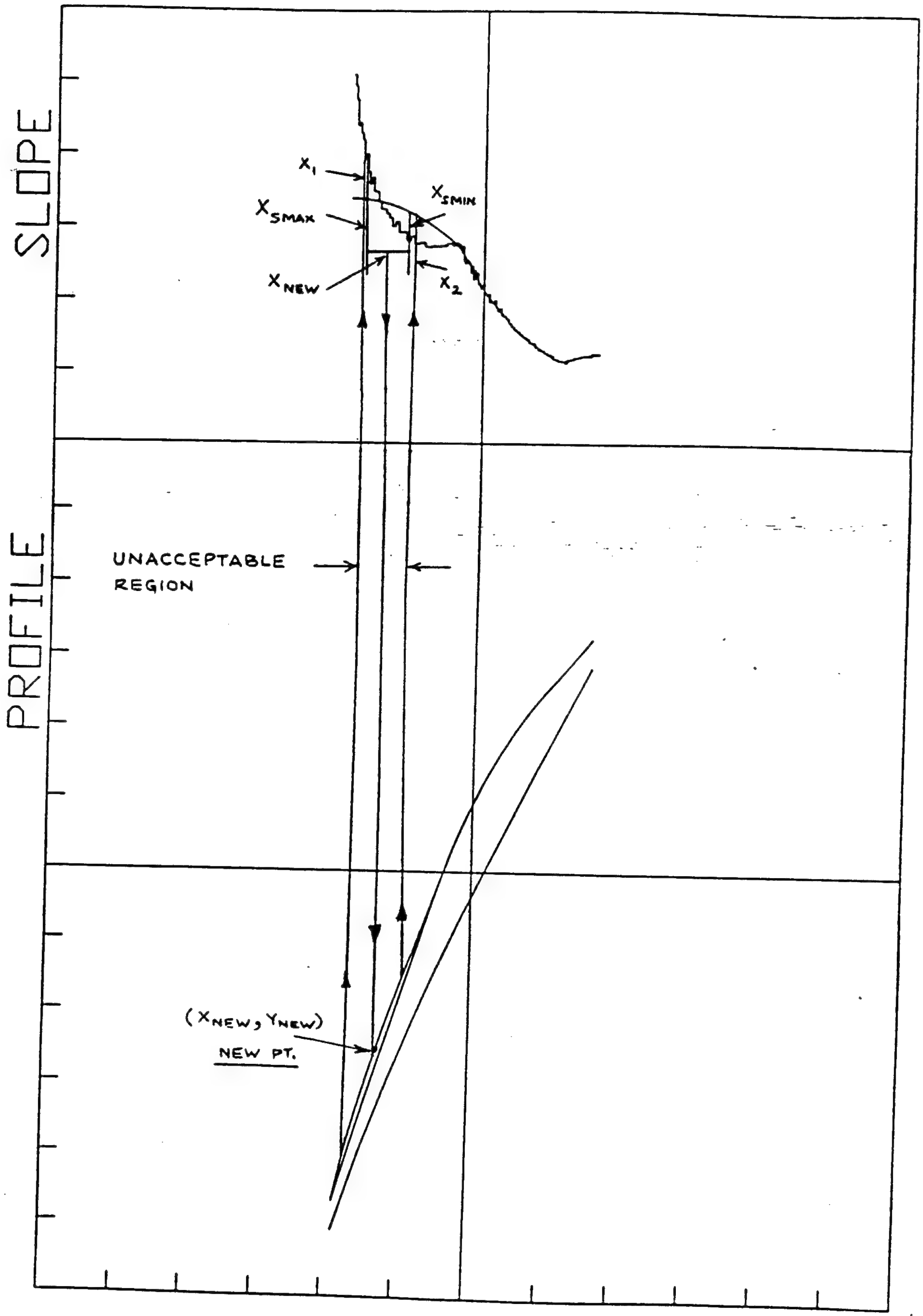


Figure 21 Determination of X(new) from Slope Data

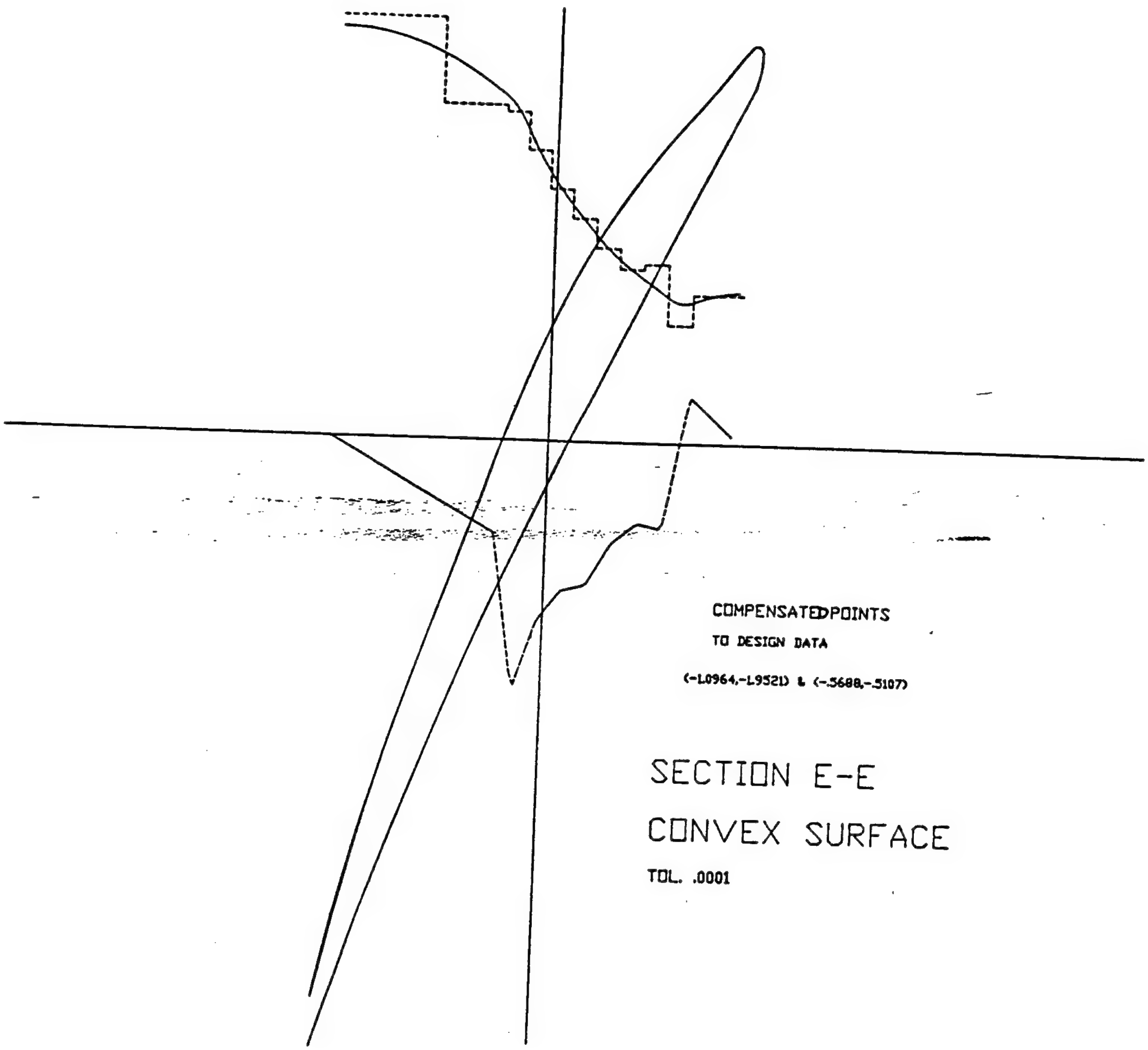


Figure 22 Illustration of Profile and Slope Matching with [X(new),Y(new)]

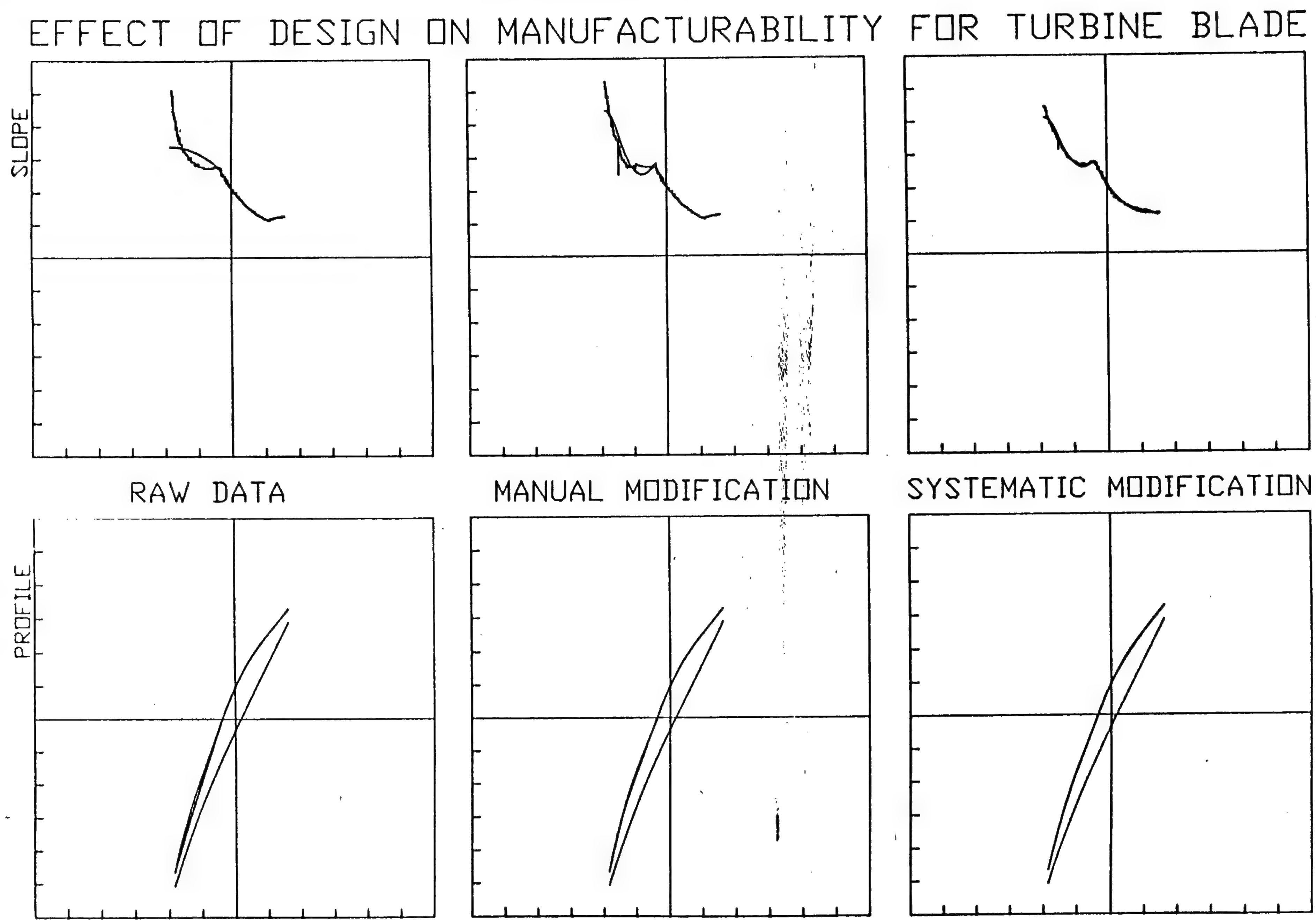


Figure 23 Illustration of the Result of Algorithm Section E-E (Tol=0.0001 in.)

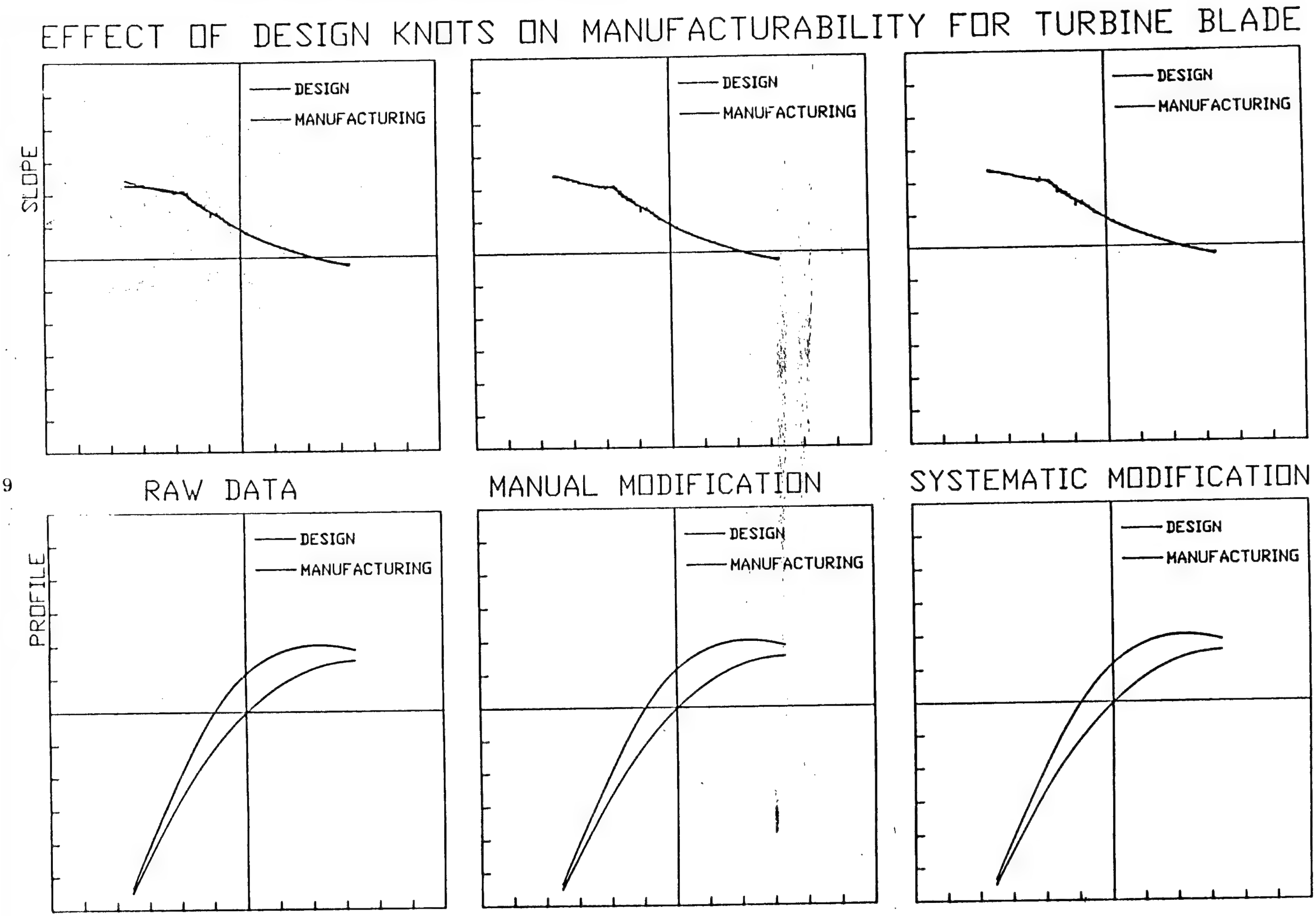


Figure 24 Illustration of the Result of Algorithm Section K-K (Tol=0.0001 in.)

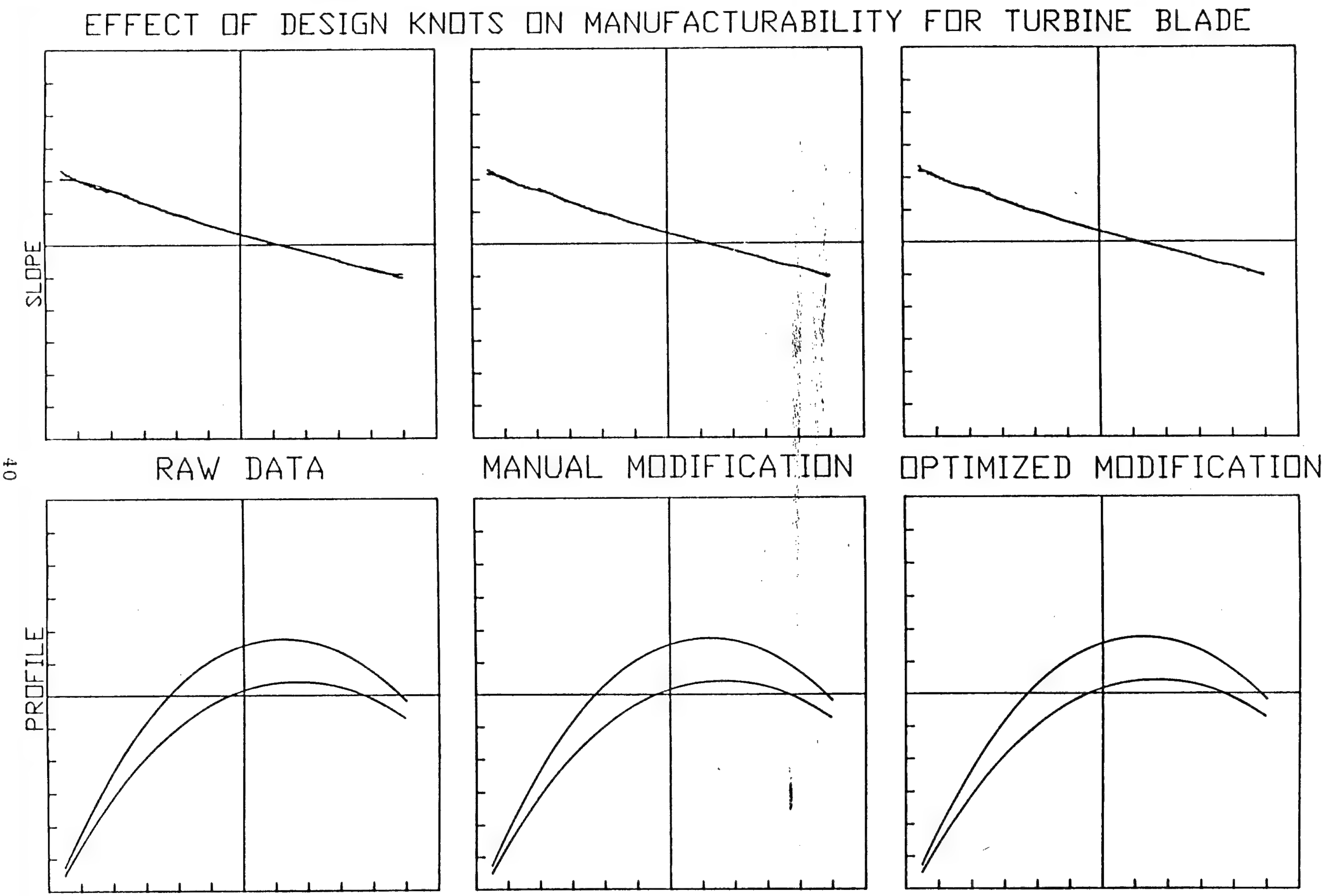


Figure 25 Illustration of the Result of Algorithm Section R-R (Tol=0.0001 in.)

CHAPTER VI
CONCLUSIONS AND REMARKS

This report illustrates the effect of design data on the manufacturability of spline geometries. Conclusions can be drawn about how the design knots affect the manufactured profile along various types of curves. The section of the turbine blade near the tip represents a curve with near straightline sections and those closer to the root represent pure spline curves.Conclusions may also be drawn about the effect of the specified tolerance on the profiles.

The following general conclusions are drawn from the various studies performed:

1. The deviation of the manufactured profile from the designed one is more pronounced for portions of the spline curve which approach a straight line.
2. The deviation of the manufactured profile from the designed one increases as the specified tolerance gets tighter.
3. By altering the design to suit manufacturability, it is possible to achieve a closer match of the design and the manufacturing profiles.
4. It is possible to systematically search for the appropriate knots which will suit both manufacturing and design criteria so far as the profiles are concerned.

For this work to be completely useful, it is necessary to check the other design criteria such as, the stress distribution, deflections, and thermal profiles after the profiles have been matched. The iteration process may progress until the other design conditions are met as well as the attainment of the manufacturability of the profile under consideration. This could be one of the future additions to this work.

Another future work is the optimization of the number and

location of knots to be used to define the profile to minimize the deviations of the profile, as well as the other design criteria.

It is expected that when the various design criteria are added to the manufacturability ones, the problem may not be easy to solve analytically and will rather involve heuristics. If that happens, then other techniques such as the use of Artificial Intelligence methods should be explored. Thus artificial intelligence and expert systems are being investigated at the present time to assess the feasibility of such methods.

REFERENCES

1. Horn, B. K. P., "Curve of Least Energy," ACM Trans Math Software, Vol 9, No.4, 1983, pp. 441-460.

2. Pressman, R. S., and Williams, J. E., Numerical Control and Computer-Aided Manufacturing, John Wiley & Sons, 1977, pp. 237-243.

3. Cohen, H., Rogers, G. F. C., and Saravana-Mutto, H. I. H., Gas Turbine Theory, 2nd ed., Longman, 1972, pp. 182-240.

4. IBM System/360 APT Numerical Control Processor (360A-CN-10X) version 4. Part Programming Manual, 1972, pp. 42-44.

5. Greville, T. N. E., "Spline Functions, Interpolation, and Numerical Quadrature," Comp. Appl. Math., Vol 3, 1982, pp. 156-168.

6. Lee, E. T. Y., "Simplified B-Spline Computational Routine," Computing, Vol 29, No.4, 1982, pp. 365-371.

7. de Boor, C., "On Calculating B-Splines," Journal of Approx. Theory, Vol 6, 1972, pp. 50-62.

8. Braibant, V. "Shape Optimal Design Using B-Splines," Comput. Methods, Appl. Mech. Eng., Vol 44, No.3, 1984, pp. 247-267.

9. Bjoerkenstam, U. C., "Curve Fitting Using Variable Segment Stiffness for Computer-Aided Design," Comput. Mech. Eng., Vol 2, No.3, 1983, pp. 61-66.

10. Fredericks, D. A., "Algebraic Method to Determine if a Point is on a Spline," IEEE Comput. Graphics Appl., Vol 4, No.2, 1984, pp. 52-53.

11. Pachner, J. Handbook of Numerical Analysis Applications, McGraw-Hill, 1984, pp. 2.1-2.13.

12. Faux, I. D., and Pratt, M. J., Computational Geometry for Design and Manufacture, Halsted Press, NY, 1979.

13. Mettke, H. "Convex Cubic Hermite-Spline Interpolation," J. Comput. Appl. Math, Vol 9, No.3, 1983, pp. 205-211.

14. Schumaker, L. L., "Splines and Computer-Aided Design," Delft Prog. Rep., Vol 9, No.4, 1984, pp. 292-304.

15. Koren, Y. Computer Control of Manufacturing Systems, McGraw-Hill, 1983, pp. 115-141.

16. Fowler, A. H., and Wilson, C. W., Cubic Spline-A Curve Fitting Routine, Report No.Y-1400, Union Carbide Corp., Nuclear Division, 1966.

17. Hummel, R., "Sampling for Spline Reconstruction", SIAM J. Appl. Math, Vol 43, No.2, 1983, pp. 278-288.
n, 1966.

APPENDIX

This appendix contains the following:

1. Sample of manufacturing data generated for various tolerances.

2. Sample of data generated by the spline routine, KNOTS.

File: data
Report: DESIGN & MFG DATA

No.	DESIGN DATA X	DESIGN DATA Y	MFG. T=0.1 X	MFG. T=0.1 Y	MFG. T=0.01 X	MFG. T=0.01 Y	MFG. T=0.001 X	MFG. T=0.001 Y	MFG. T=0.0001 X	MFG. T=0.0001 Y
1	-1.857	-4.666	-1.857	-4.666	-1.857	-4.666	-1.857	-4.666	-1.857	-4.666
2	-0.438	-0.137	-0.986	-1.646	-1.635	-3.696	-1.794	-4.358	-1.838	-4.569
3	-0.246	0.381	-0.438	-0.137	-1.311	-2.601	-1.721	-4.039	-1.818	-4.470
4	-0.052	0.844	-0.246	0.381	-0.842	-1.246	-1.638	-3.707	-7.797	-4.370
5	0.147	1.256	-0.052	0.844	-0.438	-0.137	-1.542	-3.360	-1.774	-4.269
6	0.347	1.622	0.147	1.256	-0.246	0.381	-1.434	-2.993	-1.751	-4.167
7	0.552	1.948	0.347	1.622	-0.052	0.844	-1.310	-2.599	-1.727	-4.064
8	0.758	2.241	0.552	1.948	0.147	1.256	-1.167	-2.168	-1.702	-3.959
9	0.966	2.506	0.758	2.241	0.347	1.622	-0.998	-1.681	-1.675	-3.853
10	1.176	2.751	0.966	2.506	0.552	1.948	-0.785	-1.089	-1.648	-3.745
11	1.600	3.271	1.176	2.751	0.758	2.241	-0.449	-0.170	-1.619	-3.636
12	1.600	2.866	1.600	3.271	0.966	2.506	-0.438	-0.137	-1.588	-3.525
13	1.243	2.162	1.600	2.866	1.176	2.751	-0.302	0.236	-1.557	-3.411
14	0.906	1.489	1.243	2.162	1.600	3.271	-0.246	0.381	-1.524	-3.296
15	0.569	0.814	0.906	1.489	1.600	2.866	-0.183	0.540	-1.490	-3.178
16	0.232	0.128	0.569	0.814	1.243	2.162	-0.125	0.678	-1.454	-3.058
17	-0.104	-0.577	0.232	0.128	0.906	1.489	-0.052	0.844	-1.416	-2.935
18	-0.437	-1.309	-0.104	-0.577	0.569	0.814	0.008	0.975	-1.377	-2.809
19	-0.768	-2.079	-0.437	-1.309	0.232	0.128	0.067	1.097	-1.336	-2.679
20	-1.097	-2.894	-0.768	-2.029	-0.104	-0.577	0.147	1.256	-1.293	-2.545
21	-1.421	-3.764	-1.097	-2.894	-0.437	-1.309	0.213	1.382	-1.247	-2.407
22	-1.857	-5.073	-1.421	-3.764	-0.768	-2.079	0.273	1.492	-1.200	-2.263
23			-1.857	-5.073	-1.097	-2.894	0.347	1.622	-1.149	-2.114
24					-1.421	-3.764	0.467	1.818	-1.095	-1.956
25					-1.772	-4.816	0.552	1.948	-1.036	-1.790

file: data
Report; DESIGN & MFG DATA

	MFG. T=0.01 X	Y	MFG. T=0.001 X	Y	MFG. T=0.0001 X	Y
26	-1.857	-5.073	0.684	2.139	-0.974	-1.612
27			0.758	2.241	-0.904	-1.418
28			0.891	2.413	-0.826	-1.201
29			0.966	2.505	-0.731	-0.942
30			1.068	2.626	-0.599	-0.581
31			1.176	2.751	-0.518	-0.359
32			1.429	3.057	-0.438	-0.137
33			1.600	3.271	-0.395	-0.019
34			1.600	2.866	-0.359	0.081
35			1.330	2.334	-0.334	0.150
36			1.243	2.162	-0.315	0.202
37			0.906	1.489	-0.298	0.247
38			0.569	0.814	-0.283	0.288
39			0.232	0.128	-0.268	0.326
40			0.063	-0.224	-0.246	0.381
41			-0.104	-0.577	-0.226	0.432
42			-0.242	-0.875	-0.207	0.480
43			-0.437	-1.309	-0.189	0.525
44			-0.658	-1.817	-0.170	0.570
45			-0.768	-2.079	-0.153	0.612
46			-0.872	-2.331	-0.135	0.653
47			-1.070	-2.826	-0.118	0.694
48			-1.097	-2.890	-0.101	0.733
49			-1.194	-3.147	-0.085	0.771
50			-1.286	-3.392	-0.052	0.844

file:data
REPORT: DESIGN & MFG DATA

	MFG. T=0.001		MFG. T=0.000	
	X	Y	X	Y
51	-1.421	-3.764	-0.033	0.883
52	-1.536	-4.095	-0.014	0.926
53	-1.676	-4.521	0.004	0.966
54	-1.857	-5.073	0.041	1.048
55			0.077	1.118
56			0.113	1.189
57			0.147	1.256
58			0.167	1.296
59			0.187	1.334
60			0.207	1.371
61			0.226	1.406
62			0.244	1.440
63			0.263	1.474
64			0.281	1.506
65			0.299	1.538
66			0.316	1.568
67			0.347	1.622
68			0.384	1.684
69			0.422	1.746
70			0.461	1.808
71			0.500	1.869
72			0.540	1.930
73			0.522	1.948
74			0.593	2.009
75			0.634	2.069

file:data
REPORT:DESIGN & MFG DATA

	MFG. T=0.0001 X	Y
76	0.676	2.128
77	0.718	2.187
78	0.758	2.241
79	0.800	2.296
80	0.843	2.352
81	0.888	2.409
82	0.936	2.469
83	0.966	2.506
84	0.998	2.544
85	1.036	2.589
86	1.086	2.647
87	1.131	2.699
88	1.154	2.725
89	1.176	2.751
90	1.257	2.847
91	1.337	2.944
92	1.416	3.041
93	1.495	3.138
94	1.573	3.237
95	1.600	3.271
96	1.644	2.952
97	1.600	2.866
98	1.514	2.698
99	1.415	2.502
100	1.285	2.245

101	1.243	2.162
102	1.118	1.912
103	0.946	1.568
104	0.906	1.489
105	0.758	1.193
106	0.663	1.004
107	0.569	0.814
108	0.490	0.654
109	0.424	0.521
110	0.367	0.403
111	0.314	0.295
112	0.232	0.128
113	0.155	-0.032
114	0.099	-0.148
115	0.051	-0.249
116	0.008	-0.840
117	-0.032	-0.425
118	-0.104	-0.577
119	-0.147	-0.671
120	-0.189	-0.762
121	-0.229	-0.849
122	-0.268	-0.983
123	-0.305	-1.014
124	-0.341	-1.094
125	-0.376	-1.171
126	-0.437	-1.309
127	-0.508	-1.469
128	-0.577	-1.628

129	-0.644	-1.784
130	-0.710	-1.940
131	-0.768	-2.079
132	-0.834	-2.238
133	-0.898	-2.896
134	-0.961	-2.551
135	-1.022	-2.705
136	-1.082	-2.857
137	-1.097	-2.894
138	-1.158	-3.054
139	-1.218	-3.211
140	-1.275	-3.364
141	-1.331	-3.515
142	-1.385	-3.663
143	-1.421	-3.764
144	-1.453	-3.869
145	-1.497	-3.980
146	-1.538	-4.102
147	-1.583	-4.237
148	-1.635	-4.395
149	-1.702	-4.601
150	-1.779	-4.837
151	-1.814	-4.942
152	-1.857	-5.073

values for convex surface

values at matching points
101ARGUMENTS OUT OF RANGE

arguments	y	slope	curvature
-1.857200	-4.666500	3.371406	.000000
-1.822627	-4.549944	3.371084	-.018618
-1.788054	-4.433411	3.370119	-.037235
-1.753481	-4.316921	3.368509	-.055853
1.718908	-4.200499	3.366256	-.071471
-1.684335	-4.084165	3.363360	-.093088
-1.649762	-3.967944	3.359820	-.111706
-1.615189	-3.851855	3.355636	-.130324
-1.580616	-3.735922	3.350809	-.148941
-1.546043	-3.620167	3.345337	-.167559
-1.511470	-3.504613	3.339222	-.186177
-1.476897	-3.389281	3.332464	-.204794
-1.442324	-3.274194	3.325062	-.223412
-1.407751	-3.159374	3.317016	-.242030
-1.373178	-3.044843	3.308326	-.260647
-1.338605	-2.930623	3.298993	-.279265
-1.304032	-2.816738	3.289016	-.297883
-1.269459	-2.703208	3.278396	-.316500
-1.234886	-2.590057	3.267131	-.335118
-1.200313	-2.477307	3.255224	-.353736
-1.165740	-2.364979	3.242672	-.372353
-1.131167	-2.253096	3.229477	-.390971
-1.096594	-2.141681	3.215638	-.409589
-1.062021	-2.030755	3.201155	-.428206
-1.027448	-1.920341	3.186029	-.446824
-.992875	-1.810461	3.170259	-.465442
-.958302	-1.701138	3.153846	-.484059
-.923729	-1.592393	3.136789	-.502677
-.889156	-1.484249	3.119088	-.521295
-.854583	-1.376728	3.100743	-.539912
-.820010	-1.269853	3.081755	-.558530
-.785437	-1.163644	3.062123	-.577148
-.750864	-1.058126	3.041847	-.595765
-.716291	-.953320	3.020928	-.614383
-.681701	-.849248	2.999365	-.633001
-.647145	-.745934	2.977159	-.651618
-.612572	-.643397	2.954308	-.670236
-.577999	-.541662	2.920815	-.688854
-.543426	-.440751	2.906677	-.707471
-.508853	-.340685	2.881896	-.726089
-.474280	-.241486	2.856471	-.744707
-.439707	-.143178	2.830402	-.763324
-.405134	-.045819	2.800376	.985456
-.370561	.050363	2.762249	-1.220128
-.335988	.145086	2.716009	-1.454801

-.301415	.238070	2.661656	-1.689474
-.266842	.329035	2.599189	-1.924147
-.232269	.417705	2.529598	-2.019854
-.197696	.503974	2.461545	-1.916925
-.163123	.587952	2.397050	-1.813996
-.128550	.669762	2.336114	-1.711067
-.093977	.749526	2.278736	-1.608138
-.059404	.827368	2.224917	-1.505209
-.024831	.903406	2.174033	-1.448246
.009742	.977712	2.124731	-1.403810
.044315	1.050340	2.076965	-1.359374
.078888	1.121343	2.030735	-1.314938
.113461	1.190775	1.986042	-1.270502
.148034	1.258688	1.942884	-1.227547
.182607	1.325127	1.900595	-1.218821
.217180	1.390110	1.858608	-1.210096
.251753	1.453646	1.816922	-1.201370
.286326	1.515746	1.775538	-1.192644
.320899	1.576421	1.734455	-1.183918
.355472	1.635680	1.693723	-1.163241
.390045	1.693554	1.654531	-1.103952
.424618	1.750108	1.617389	-1.044663
.459191	1.805414	1.582297	-.985375
.493764	1.859542	1.549255	-.926086
.528337	1.912562	1.518262	-.866797
.562910	1.964547	1.489262	-.817663
.597483	2.015552	1.461477	-.789691
.632056	2.065613	1.434658	-.761719
.666629	2.114764	1.408807	-.733748
.701202	2.163038	1.383922	-.705776
.735775	2.210468	1.360005	-.677804
.770348	2.257087	1.336982	-.662072
.804921	2.302914	1.313959	-.669814
.839494	2.347939	1.290668	-.677555
.874067	2.392155	1.267109	-.685297
.908640	2.435552	1.243282	-.693038
.943213	2.478120	1.219188	-.700780
.977786	2.519852	1.195181	-.646123
1.012359	2.560823	1.175984	-.464395
1.046932	2.601239	1.163070	-.282668
1.081505	2.641317	1.156439	-.100942
1.116078	2.681274	1.156090	.080785
1.150651	2.721328	1.162025	.262513
1.185224	2.761695	1.173984	.387751
1.219797	2.802509	1.186831	.355439
1.254370	2.843747	1.198562	.323126
1.288943	2.885372	1.209174	.290813
1.323516	2.927344	1.128670	.258501
1.358089	2.969625	1.227049	.226188
1.392662	3.012177	1.234310	.193875
1.427235	3.054960	1.240454	.161563
1.461808	3.097936	1.245481	.129250
1.496381	3.141067	1.1249391	.096938
1.530954	3.184314	1.252184	.064625
1.565527	3.227638	1.253860	.032312

1.600100	.000000	.000000	.000000

VALUES AT DATA POINTS ARE

arguments	y	slope	curvature
-1.857200	-4.666500	3.371406	.000000
-.437700	-.137500	2.828869	-.764405
-.246500	.381500	2.558644	-2.062222
-.052000	.843800	2.213855	-1.483166
.146600	1.255900	1.944645	-1.227909
.347300	1.621800	1.703287	-1.177255
.551700	1.947800	1.498479	-.826732
.758500	2.241200	1.344811	-.659419
.966400	2.506200	1.202879	-.705972
1.176100	2.751000	1.170407	.396279
1.600100	3.271000	1.254419	.000000

values for concave surface

values at marching points

101ARGUMENT OUT OF RANGE

arguments	y	slope	curvature
-1.857200	-5.072900	3.085187	.000000
-1.822627	-4.966254	3.083599	-.091875
-1.788054	-4.859718	3.078835	-.183749
-1.753481	-4.753401	3.070894	-.275624
-1.718908	-4.647414	3.059776	-.367498
-1.684335	-4.541867	3.045482	-.459373
-1.649762	-4.436868	3.028012	-.551248
-1.615189	-4.332528	3.007366	-.643122
-1.580616	-4.228957	2.983543	-.734997
-1.546043	-4.126265	2.956544	-.826872
-1.511470	-4.024560	2.926368	-.918746
-1.476897	-3.923955	2.893016	-1.010621
-1.442324	-3.824557	2.856488	-1.102496
-1.407751	-3.726475	2.817197	-1.131855
-1.373178	-3.629738	2.779298	-1.060599
-1.338605	-3.534269	2.743861	-.989343
-1.304032	-3.439983	2.710888	-.918086
-1.269459	-3.346794	2.680379	-.846830
-1.234886	-3.254616	2.652333	-.775574
-1.200313	-3.163367	2.626751	-.704317
-1.165740	-3.072959	2.603633	-.633061
-1.131167	-2.983308	2.582978	-.561805
-1.096594	-2.894328	2.564786	-.490770
-1.062021	-2.805949	2.547800	-.491876
01.027448	-2.718156	2.530775	-.492982

-.992875	-2.630957	2.513712	-.494088
-.958302	-2.544346	2.496611	-.495194
-.923729	-2.458327	2.479471	-.496300
-.889156	-2.372900	2.462293	-.497406
-.854583	-2.288069	2.445078	-.498512
-.820010	-2.203834	2.427823	-.499618
-.785437	-2.120195	2.410531	-.500724
-.750864	-2.037155	2.393260	-.494962
-.716291	-1.954706	2.376363	-.482529
-.681718	-1.872834	2.359895	-.470095
-.647145	-1.791524	2.343858	-.457661
-.612572	-1.710761	2.328250	-.445228
-.577999	-1.630530	2.313072	-.432794
-.543426	-1.550816	2.298324	-.420360
-.508853	-1.471605	2.284006	-.407927
-.474280	-1.392881	2.270118	-.395493
-.439707	-1.314631	2.256659	-.383059
-.405134	-1.236839	2.243515	-.377831
-.370561	-1.159499	2.230533	-.373143
-.335988	-1.082605	2.217713	-.368454
-.301415	-1.006151	2.205056	-.363765
-.266842	-.930132	2.192560	-.359076
-.232269	-.854542	2.180227	-.354387
-.197696	-.779376	2.168056	-.349698
-.163123	-.704628	2.156047	-.345010
-.128550	-.630292	2.144200	-.340321
-.093977	-.556363	2.132553	-.327805
-.059404	-.482823	2.121787	-.294997
-.024831	-.409637	2.112155	-.262189
.009742	-.336763	2.103657	-.229381
.044315	-.264164	2.096294	-.196573
.078888	-.191800	2.090065	-.163766
.113461	-.119631	2.084970	-.130958
.148034	-.047619	2.081010	-.098150
.182607	.024275	2.078183	-.065342
.217180	.096092	2.076492	-.032534
.251753	.167866	2.075432	-.051914
.286326	.239577	2.072584	-.112820
.320899	.311153	2.067631	-.173727
.355472	.382521	2.060571	-.234633
.390045	.453609	2.051407	-.295540
.424618	.524343	2.040136	-.356446
.459191	.594652	2.026760	-.417352
.493764	.664462	2.011278	-.478259
.528337	.733699	1.993690	-.539165
.562910	.802293	1.973997	-.600071
.597483	.870203	1.955755	-.410281
.632056	.937623	1.945796	-.165819
.666629	1.004844	1.944290	.078641
.701202	1.072160	1.951234	.323103
.735775	1.139862	1.966631	.167565
.770348	1.208242	1.990479	.812025
.804921	1.277593	2.022779	1.056487
.839494	1.348207	2.063531	1.300949
.874067	1.420375	2.112734	1.545410

.908640	1.494391	2.170295	1.712129
.943213	1.570277	2.214675	.855153
.977786	1.647185	2.229425	-.001817
1.012359	1.724092	2.214549	-.858793
1.046932	1.799971	2.170043	-1.715769
1.081505	1.873800	2.095911	-2.572739
1.116078	1.944553	1.992149	-3.429714
1.150651	2.011208	1.858759	-4.286690
1.185224	2.072738	1.695742	-5.143660
1.219797	2.128120	1.503096	-6.000636
1.254370	2.176340	1.283518	-6.374762
1.288943	2.217032	1.074143	-5.737284
1.323516	2.250866	.886808	-5.099810
1.358089	2.278605	.721512	-4.462332
1.392662	2.301010	.578255	-3.824853
1.427235	2.318843	.457038	-3.187380
1.461808	2.332866	.357861	-2.549901
1.496381	2.343842	.280723	-1.912424
1.530954	2.352531	.225624	-1.274950
1.565527	2.359697	.192565	-.637472
1.600100	.000000	.000000	.000000

VALUES AT DATA POINTS ARE

arguments	y	slope	curvature
-1.857200	-5.072900	3.085187	.000000
-1.421000	-3.763900	2.832374	-1.159162
-1.096700	-2.894600	2.564838	-.490767
-.768400	-2.079200	2.401995	-.501269
-.437300	-1.309200	2.255738	-.382194
-.103600	.576900	2.135751	-.336937
.232500	.127900	2.076105	-.017996
.569100	.814500	1.970249	-.610976
.906200	1.489100	2.166043	1.772617
1.243200	2.161600	1.355877	-6.580728
1.600100	2.366100	.181546	.000000

AREA	X-CENTROID	Y-CENTROID	I - XX	I - YY
.3204E+01	-.2877E-01	-.3619E-01	.1213E+02	.2380E+01

values at data pts.

convex surface

X	Y	SLOPE
-1.857000	-4.666000	3.467279
-.986000	-1.646000	3.467279
-.438000	.137000	2.753650
-.246000	.381000	2.697917
-.052000	.844000	2.386598
.147000	1.256000	2.070352
.347000	1.622000	1.830000
.552000	1.948000	1.590244
.758000	2.241000	1.422330
.966000	2.506000	1.274039
1.176000	2.751000	1.166666
1.600000	3.271000	1.226415

concave surface

X	Y	SLOPE
-1.857000	-5.073000	3.002294
-1.421000	-3.764000	3.002294
-1.097000	-2.894000	2.685185
-.768000	-2.079000	2.477204
-.437000	-1.309000	2.326284
-.104000	-.577000	2.198198
.232000	.128000	2.098214
.569000	.814000	2.035608
.906000	1.489000	2.002967
1.243000	2.162000	1.997033
1.600000	2.866000	1.971989

AREA	X-CENTROID	Y-CENTROID	I - XX	I - YY
.3235E+01	-.8441E-01	-.1334E+00	.1241E+02	.2373E+01

values at data pts.

convex surface

X	Y	SLOPE
-1.857000	-4.666000	4.369369
-1.635000	-3.696000	4.369369
-1.311000	-2.601000	3.379630
-.842000	-1.246000	2.889126
-.438000	.137000	2.745050
-.246000	.381000	2.697917
-.052000	.844000	2.386598
.147000	1.256000	2.070352
.347000	1.622000	1.830000
.552000	1.948000	1.590244
.758000	2.241000	1.422330
.966000	2.506000	1.247039
1.176000	2.751000	1.166666
1.600000	3.271000	1.226415

concave surface

X	Y	SLOPE
-1.857000	-5.073000	3.023528
-1.772000	-4.816000	3.023528
-1.421000	-3.764000	2.997152
-1.097000	-2.894000	2.685185
-.768000	-2.079000	2.477204
-.437000	-1.309000	2.326284
-.104000	-.577000	2.198198
.232000	.128000	2.098214
.569000	.814000	2.035608
.906000	1.489000	2.002967
1.243000	2.162000	1.997033
1.600000	2.866000	1.971989

AREA	X-CENTROID	Y-CENTROID	I - XX	I - YY
.3342E+01	-.1285E+00	-.2322E+00	.1356E+02	.2605E+01

value at data pts.

convex surface

x	y	slope
-1.857000	-4.666000	4.888893
-1.794000	-4.358000	4.888893
-1.172100	-4.039000	4.369855
-1.638000	-3.707000	4.000003
-1.542000	-3.360000	3.614586
-1.434000	-2.993000	3.398146
-1.310000	-2.599000	3.177418
-1.167000	-2.168000	3.013988
-.998000	-1.681000	2.881656
-.785000	-1.089000	2.779343
-.449000	-.170000	2.735119
-.438000	-.137000	2.999999
-.302000	.236000	2.742647
-.246000	.381000	2.589287
-.183000	.540000	2.523809
-.125000	.678000	2.379310
-.052000	.844000	2.273973
.008000	.975000	2.183334
.067000	1.097000	2.067796
.147000	1.256000	1.987501
.213000	1.382000	1.909090
.273000	1.492000	1.833333
.347000	1.622000	1.756757
.467000	1.818000	1.633333
.552000	1.948000	1.529412
.684000	2.139000	1.446969
.758000	2.241000	1.378378
.891000	2.413000	1.293235
.966000	2.506000	1.239998
1.068000	2.626000	1.176470
1.176000	2.751000	1.157407
1.429000	3.057000	1.209486
1.600000	3.271000	1.251462

concave surface

x	y	slope
-1.857000	-5.073000	3.049724
-1.676000	-4.521000	3.049724
-1.536000	-4.095000	3.042858
-1.421000	-3.764000	2.878259
-1.286000	-3.392000	2.755556
-1.194000	-3.147000	2.663042
-1.097000	-2.890000	2.649484

values at data points

convex surface

x	y	SLOPE
-1.857000	-4.666000	5.105287
-1.838000	-4.659000	5.105287
-1.818000	-4.470000	4.949974
-1.797000	-4.370000	4.761921
-1.774000	-4.269000	4.391297
-1.751000	-4.167000	4.434793
-1.727000	-4.064000	4.291465
-1.702000	-3.959000	4.200005
-1.675000	-3.853000	3.925923
-1.648000	-3.745000	4.000009
-1.619000	-3.636000	3.758614
-1.588000	-3.525000	3.580651
-1.557000	-3.411000	3.677420
-1.524000	-3.296000	3.484850
-1.490000	-3.178000	3.470585
-1.454000	-3.058000	3.333329
-1.416000	-2.935000	3.236847
-1.377000	-2.809000	3.230765
-1.336000	-2.679000	3.170734
-1.293000	-2.545000	3.116276
-1.247000	-2.407000	3.000000
-1.200000	-2.263000	3.063835
-1.149000	-2.114000	2.921657
-1.095000	-1.956000	2.925927
-1.036000	-1.790000	2.813559
-0.974000	-1.612000	2.870966
-0.904000	-1.418000	2.771429
-0.826000	-1.201000	2.782051
-0.731000	-0.942000	2.726317
-0.599000	-0.581000	2.734848
-0.518000	-0.359000	2.740741
-0.438000	-0.137000	2.775000
-0.395000	-0.019000	2.744187
-0.359000	0.081000	2.777777
-0.334000	0.150000	2.760000
-0.315000	0.202000	2.736843
-0.298000	0.247000	2.647060
-0.283000	0.288000	2.733330
-0.268000	0.326000	2.533337
-0.246000	0.381000	2.500000
-0.226000	0.432000	2.549999
-0.207000	0.480000	2.526315
-0.189000	0.525000	2.499998
-0.170000	0.570000	2.368423
-0.153000	0.612000	2.470587
-0.135000	0.653000	2.277779

-0.118000	0.694000	2.411765
-0.101000	0.733000	2.294116
-0.085000	0.771000	2.375003
-0.052000	0.844000	2.212120
-0.033000	0.885000	2.157895
-0.014000	0.926000	2.157895
0.004000	0.966000	2.222223
0.041000	1.043000	2.081080
0.077000	1.118000	2.083335
0.113000	1.189000	1.972222
0.147000	1.256000	1.970589
0.167000	1.296000	1.999998
0.187000	1.334000	1.899998
0.207000	1.371000	1.850003
0.226000	1.406000	1.842104
0.244000	1.440000	1.888890
0.263000	1.474000	1.789469
0.281000	1.506000	1.777784
0.299000	1.530000	1.777774
0.316000	1.568000	1.764702
0.347000	1.622000	1.741937
0.384000	1.684000	1.675677
0.422000	1.746000	1.631580
0.461000	1.808000	1.589741
0.500000	1.869000	1.564102
0.540000	1.930000	1.524999
0.552000	1.948000	1.500005
0.593000	2.009000	1.487807
0.634000	2.069000	1.463413
0.676000	2.128000	1.404762
0.718000	2.187000	1.404762
0.758000	2.241000	1.349997
0.800000	2.296000	1.309525
0.843000	2.352000	1.302326
0.888000	2.409000	1.266665
0.936000	2.469000	1.250004
0.966000	2.506000	1.233330
0.998000	2.544000	1.187496
1.036000	2.589000	1.184213
1.086000	2.647000	1.160003
1.131000	2.699000	1.155549
1.154000	2.725000	1.130436
1.176000	2.751000	1.181822
1.257000	2.847000	1.185185
1.337000	2.944000	1.212501
1.416000	3.041000	1.227847
1.495000	3.138000	1.227850
1.573000	3.237000	1.269231
1.600000	3.271000	1.259253

concave surface

x	y	SLOPE
-1.857000	-5.073000	3.046514
-1.814000	-4.942000	3.046514
-1.779000	-4.837000	3.000003
-1.702000	-4.601000	3.064935
-1.635000	-4.395000	3.074623
-1.583000	-4.237000	3.038459
-1.538000	-4.102000	2.999997
-1.497000	-3.980000	2.975615
-1.458000	-3.868000	2.871792
-1.421000	-3.764000	2.810818
-1.385000	-3.663000	2.805550
-1.331000	-3.515000	2.740740
-1.275000	-3.364000	2.696429
-1.218000	-3.211000	2.684216
-1.158000	-3.054000	2.616665
-1.097000	-2.894000	2.622949
-1.082000	-2.857000	2.466665
-1.022000	-2.705000	2.533334
-0.961000	-2.551000	2.524590
-0.898000	-2.396000	2.460320
-0.834000	-2.238000	2.468750
-0.768000	-2.079000	2.409090
-0.710000	-1.940000	2.396549
-0.644000	-1.784000	2.363637
-0.577000	-1.628000	2.328359
-0.508000	-1.469000	2.304348
-0.437000	-1.309000	2.253520
-0.376000	-1.171000	2.262295
-0.341000	-1.094000	2.200001
-0.305000	-1.014000	2.222221
-0.268000	-0.933000	2.189190
-0.229000	-0.849000	2.153847
-0.189000	-0.762000	2.174999
-0.147000	-0.671000	2.166667
-0.104000	-0.577000	2.186046
-0.032000	-0.425000	2.111111
0.008000	-0.340000	2.125000
0.051000	-0.249000	2.116279
0.099000	-0.148000	2.104167
0.155000	-0.032000	2.071429
0.232000	0.128000	2.077922
0.314000	0.295000	2.036585
0.367000	0.403000	2.037736
0.424000	0.521000	2.070177
0.490000	0.654000	2.015150
0.569000	0.814000	2.025317
0.663000	1.004000	2.021276
0.758000	1.193000	1.989473
0.906000	1.489000	2.000000
0.946000	1.568000	1.975002
1.118000	1.912000	1.999999

1.243000	2.162000	2.000000
1.285000	2.245000	1.976192
1.415000	2.502000	1.976925
1.514000	2.698000	1.979795
1.600000	2.866000	1.953489

AREA	X-CENTRIOD	Y-CENTROID	I-XX	I-YY
.3353E+01	-.1330E+00	-.2390E+00	.1365E+02	.2627E+01